Interest-bearing Money, Illiquid Bonds, and Banking in a News Economy

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Abstract

Private currencies can facilitate intertemporal exchange under limited commitment, but they may exhibit excessive price volatility when backed by productive assets whose expected short-run return is subject to news shocks or flows of information. In the model, banks act as intermediaries by supplying private currencies in the form of bank deposits, backed by short-run expected returns on firms' output pledged as collateral. Adverse news events about firm productivity in the short run can induce price volatility in bank deposits, potentially leading to a liquidity shortage. Adding household heterogeneity with news shocks results in bank deposits being priced at a premium in economies where liquidity matters, and in particular, when there is a liquidity shortage. Interest-bearing money, not backed by productive assets, can help alleviate this shortage. I show how interest-bearing money offers an additional policy tool that can help lift depressed asset prices. The interest rate affects asset prices via the investment channel, with banks using interest-bearing reserves as insurance against risk shocks. The relative insensitivity of the value of money to news events means social welfare could be improved, even if lump-sum taxation is not permitted. However, I find that the power of interest-bearing money is only limited to assuming that household preferences are publicly observable, so that type-contingent transfers are permissible. Extending the model, an illiquid bond and a cash-in-advance constraint enable the coexistence of government debt and private currencies in the absence of news.

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1 Introduction

Central banks and government-issued fiat currencies are relatively recent developments of the twentieth century. Private banks were already present in ancient Mesopotamia and Egypt, predating the introduction of coinage by a few centuries. (Davies (2010)). Even in the relatively recent 1800s, many countries, including the United States, Australia, Canada, and Scotland, relied on private currencies in the form of banknotes issued by private banks. The Free Banking era in the United States (1837-1863) serves as a notable example. There is a great deal of debate about whether these monetary systems involving private monies were entirely successful (see Williamson (1999) and the references cited therein). While private currencies proved successful in countries like Canada and Scotland, they often faced challenges and were perceived as unstable in regions such as the United States (see Champ (2007)).[1] Recent technological advances in private money systems, such as gift cards and cryptocurrency, mean that these systems are now more important than ever. This then raises the question: how can private money circulate as a medium of exchange to facilitate intertemporal trade amidst such instability? In this paper, I model news shocks or flows of information as a potential source of this instability, which could lead to excessive volatility in privately-issued bank money, hindering its efficiency as a payment instrument.

The seminal work of Hirshleifer (1971) pioneered the exploration into the role of information in financial markets, emphasizing that public information is not invariably socially beneficial due to its potential to eliminate insurance possibilities. While much of the recent literature focuses on the influence of information disclosure or monetary policy announcements on markets (e.g., Andolfatto and Martin (2013), Andolfatto et al. (2014), Dang et al. (2017), Gu et al. (2020), and Choi and Liang (2021)), there has been relatively limited attention given to the impact of technological uncertainty on economies relying on exchange media when banks carry out the process of asset transformation and are deemed essential. To address this gap, I develop a general equilibrium model where banks issue their own private currencies in the form of bank deposits. Because of commitment issues, banks emerge as agents that are trustworthy and provide currency that make them essential. However, the value of bank money is susceptible to information frictions stemming from technological uncertainty, as bank deposits are backed by risky assets.

For my formal analysis, I extend the Andolfatto and Martin (2013) framework by incorporating banks and interest-bearing money while accounting for information frictions. To do this, I borrow some elements of the model from Chiu et al. (2019) framework, which extends the unified framework of Lagos and Wright (2005) by introducing an imperfect banking sector and inside money creation. In my model, households consisting of consumers and producers use exchange media to realize intertemporal gains to trade in the absence of commitment. Entrepreneurs or

[1] This was particularly evident during the early stages of development across different states, although the Suffolk system in New England during the Free Banking era stands as an exception.

firms have investment opportunities, but since they are not endowed with any resources they must borrow from the banks. Banks act as financial intermediaries between the entrepreneurs and households by creating deposits and issuing loans. Bank deposits function as exchange media that facilitate intertemporal exchange. This makes the banks essential in the model, as otherwise, there would be no gains from trade. Inside money is generated by the banks in the form of bank deposits, which also represent claims to entrepreneurs' output. The return to entrepreneurs' output is subject to aggregate risk, leading to volatility in deposit prices, akin to that in Andolfatto and Martin (2013).

To be more specific, bank deposit prices are influenced by flows of information concerning entrepreneurs' productive technology. The flows of information, referred to as "news," represent the conditional forecast over the future productivity of entrepreneurs, which in turn causes deposit prices to fluctuate. The conditional forecast of entrepreneurs' productivity is assumed to be stochastic over short horizons, while the forecast productivity over long horizons is assumed to be constant. This type of technological uncertainty is transmitted into the economy as a liquidity shortage in the equilibrium. Due to the fixed supply of bank deposits, news does not play an allocative role—it has no social value as observed in Andolfatto and Martin (2013) and Hirshleifer (1971). Negative news on productive technology can lead to binding debt-constraints, depressing economic activity. Since bank deposits are backed by the future productivity of entrepreneurs' output, this itself creates a problem for the liquidity provisioning of their use as payment instruments.

To illustrate the role of liquidity in determining asset prices, I include agent heterogeneity in the form of consumer preference heterogeneity with two types of consumers. Including consumer preference heterogeneity enables liquidity to be determined endogenously. In particular, I explicitly show how bank deposits are priced at a premium when individuals with a higher marginal utility of consumption have a pressing need to consume. Consistent with what others have found, I find that in the event of negative news shocks, the inefficiency as a result of binding debt-constraints lead to depressed asset prices.

Evidently, the presence of price volatility in private currencies does not necessarily preclude their use as exchange media, as long as efficiency can be achieved.[2] Despite the potential for excessive fluctuations in the price of bank deposits in response to news events, they can still function as exchange media to facilitate intertemporal trade when commitment is limited. This is primarily based on the assumption that news has no social value.[3] However, inefficiency in equilibria can arise when liquidity is scarce.

The private sector actively seeks ways to improve liquidity with high-quality payment

[2]When it comes to cryptocurrencies like Bitcoin and Ethereum, private currencies of this nature may inherently suffer from inefficiencies (see Fernández-Villaverde and Sanches (2019)). This inefficiency can be attributed to the self-interested nature of the private issuers, which poses challenges in achieving overall efficiency. However, in my model, there is nothing inherently wrong with the circulation of bank deposits as private currencies per se, except for their exposure to productivity shocks that are beyond the control of the private issuers.

[3]This assumption contradicts conventional wisdom but was originally demonstrated in Hirshleifer (1971), where the presence of useless information can lead to economic fluctuations that may reduce welfare.

instruments in financial markets, employing techniques such as tranching assets. For instance, banks utilize tranching in the creation of Collateralized Debt Obligations (CDOs), which involve pooling various debt instruments like bonds, loans, and mortgages. These CDOs are then divided into different tranches, each carrying a distinct risk level and cash flow priority. Senior tranches are given priority in receiving interest payments and principal repayments, while junior or equity tranches offer higher potential returns but come with increased risk exposure. Similar tranching practices are applied by banks in the bundling of individual mortgage loans into mortgage-backed securities (MBS), considering the credit quality and risk characteristics of the underlying mortgages. Asset-Backed Securities (ABS) and Collateralized Loan Obligations (CLOs) are other examples of tranching practices employed by banks. However, in my model, while the tranching of claims against the entrepreneurs' output is feasible (similar to Andolfatto and Martin (2013)), the effectiveness of tranching diminishes when there is a shortage of high-quality tranches available given a fixed supply of assets. This scenario often occurs during financial crises.

Tranching of assets is not the only means by which the private sector can enhance liquidity. Another method involves the nondisclosure of information, as highlighted in Andolfatto and Martin (2013). Private banks often favor internal "mark-to-model" methods over "mark-to-market" ones when reporting asset valuations, seeking stability and predictability. Asset prices in the market can be highly volatile, fluctuating due to a multitude of factors, many of which may be short-term and related to market sentiment. In contrast, mark-to-model provides a more stable and controlled way to value assets, relying on internal estimates and assumptions grounded in long-term fundamentals. Notably, financial regulators and central banks also engage in nondisclosure practices. For instance, the Federal Reserve collects a substantial amount of data regarding the health of private banks through its bank stress test models, yet specific operational vulnerabilities may not be disclosed publicly to prevent potential negative market reactions. Furthermore, the Fed often withholds specifics of its emergency lending operations, including details about which institutions borrowed and under what terms.[4] Regulators primarily justify these nondisclosure practices as means to promote liquidity.

If the private sector faces constraints in providing sufficient liquidity through high-quality payment instruments, central bank intervention becomes necessary to address a liquidity shortage and mitigate potential crises by providing liquid liabilities. In particular, given a fixed supply of assets, monetary policy can indeed play an important role in preventing liquidity crises, especially when there is uncertainty in asset returns.

While lump-sum tax instruments are typically assumed in monetary theory literature, I relax this assumption. Instead of issuing zero-interest-bearing fiat money, the central bank issues interest-bearing money that can be used by households as a payment instrument (see Andolfatto (2010)). Households earn interest on their money holdings, which banks exchange for deposits to meet a reserve requirement imposed by the central bank. Banks also earn interest on their

[4] See Andolfatto and Martin (2013) for more examples of nondisclosure practices.

money reserves. The absence of a lump-sum tax instrument implies that Friedman rule is not implementable. That is, deflation is not feasible. I demonstrate that the implementation of a first-best allocation requires a positive inflation and a strictly positive nominal interest rate in a news economy. The central bank can circumvent the liquidity shortage by issuing interest-bearing money. This is conditional on a critical assumption that household preferences are publicly observable, so that type-contingent money transfers are feasible. With this in mind, interest-bearing money in my paper can also be thought of as a weak form of central bank digital currency (CBDC), which has generated a lot of interest among policymakers recently. The technological innovation of such a currency may render the past actions of individuals observable to the central bank—that is, household preference shock is assumed to be public, which is why I refer to interest-bearing money here as a weak CBDC. Individuals may earn interest on their CBDC balances through central bank accounts in the same way as deposit accounts nowadays. I show that the interest rate serves as an additional policy tool to alleviate the downward pressure on asset prices caused by the increased inefficiency resulting from negative news events. Specifically, the interest rate influences asset prices through the investment channel, as banks invest in interest-bearing reserves to hedge against risk shocks.

To some extent, the welfare advantages of interest-bearing money hinge on the assumption that the central bank can observe household preferences—which is indeed a strong assumption. To address this limitation, I introduce an illiquid bond by assuming that households possess private information about their types along the lines of Kocherlakota (2003) and Andolfatto (2011). The welfare benefits of an illiquid bond market persist in a steady state, even in the presence of news shocks. Bonds play a crucial role in reallocating money towards consumer types with different intertemporal marginal rates of substitution. However, in the presence of news shocks, when both banks and entrepreneurs face binding constraints, the inefficiencies in the lending market cannot be fully resolved by the coexistence of illiquid bonds and interest-bearing money. Interestingly, in the absence of news shocks, private banks themselves can remove the inefficiencies in the lending market, making central bank intervention unwarranted. By imposing a cash-in-advance constraint and rendering bank deposits illiquid, it becomes possible to eliminate suboptimality in the lending market and create an environment where government debt instruments can coexist with private money, particularly in the absence of news shocks.

My paper contributes to the New Monetarist literature with financial intermediation by modeling news shock with an active banking sector along with interest-bearing assets. Berentsen et al. (2007) was the first to incorporate money and banking into the Lagos and Wright (2005) framework with a perfectly competitive banking sector. In my model, the banking sector is also perfectly competitive but with news shocks and includes consumer heterogeneity. In Keister and Sanches (2019), the banking sector is also perfectly competitive and CBDC—similar to interest-bearing money in this paper—competes with bank deposits, but there are no news shocks and consumer heterogeneity. Consistent with their finding, I show how bank deposits are priced at a premium when there is asset scarcity. In a related work, Hu (2021) takes on a mechanism-design approach to study the implementation of optimal policy through the interest rate on excess reserves by including a pledgeability constraint on banks. Gu et al. (2019) show that financial intermediation with banking is inherently unstable. Since interest-bearing money in my paper is very similar to CBDC, other papers that study CBDC with banking include Andolfatto (2021), Williamson (2019), Williamson (2021), Monnet et al. (2019), and Brunnermeier and Niepelt (2019). This paper is also related to transmission channels of monetary policy through the banking system; see the seminal works of Bernanke and Blinder (1992) and Bernanke et al. (1999). Other relevant papers of banking and liquidity include Drechsler et al. (2017), Goodfriend and McCallum (2007), Christiano et al. (2014), and Kiyotaki and Moore (2019). Since I consider three different types of assets, the recent paper by Amendola et al. (2021)—where money, bonds, and equity are included but without informational asymmetries—is also pertinent.

The remainder of the paper is organized as follows. Section 2 presents the environment. Sections 3 and 4 explore two regimes: one where bank deposits solely function as exchange media in a private economy, and another where bank deposits and interest-bearing money coexist as means of payment. Section 5 introduces an illiquid bond, which becomes essential when the added friction of private information over consumption patterns of households is considered. The model is further extended in this section by imposing a cash-in-advance constraint on bank deposits, rendering them illiquid and establishing the conditions for the coexistence of money, bonds, and bank deposits in equilibrium. The final section is a conclusion.

2 Environment

Time is discrete and continues forever. Each time-period t is divided into two subperiods: day and night. There are four types of agents in the economy: a unit measure of infinitely-lived households comprised of consumers and producers (divided evenly), a continuum of entrepreneurs with measure 1, and a continuum of bankers with measure 1. All agents reside in centralized locations in both subperiods (there are no search frictions).

Households belong to one of two permanent groups: Group 1 and Group 2. Each group is of equal measure. Denote by A and B the set of Group 1 and Group 2, respectively. All households have common preferences and have the ability to produce and consume the day output. Let $x_t(i) \in \mathbb{R}$ denote household consumption (production, if negative) of output (or good) during the day by household $i \in A \cup B$ at date t. Preferences are linear in $x_t(i)$, which implies that utility is transferable.

At the beginning of the night, households experience an idiosyncratic shock that determines whether they are consumers or producers with equal probability. The shock is *i.i.d.* across households and time. Consumer heterogeneity is realized at the beginning of each night after another shock, which occurs with equal probability. Let $\omega_t(i)$ denote the shock on consumer type, where $\omega_t(i) \in \{\omega_l = 1, \omega_h = \delta\}$ and $\delta > 1$.[5] This shock is *i.i.d.* across consumers within each group and across time. I will consider both public and private information structures by imposing assumptions on whether or not $\omega_t(i)$ is observable.

Denote by $\{c_t(i), y_t(i)\} \in \mathbb{R}_+^2$ the consumption and production, respectively, of the night good by household $i \in A \cup B$ at date t. The utility associated from consumption at night is given by $\omega_t(i)u(c_t(i))$, where $u'' < 0 < u'$, $u'(0) = \infty$ and $u(0) = 0$. The utility associated from production at night is given by $v(y_t(i))$, where $v' > 0$ for $y > 0$, $v'' \geq 0$ and $v(0) = 0$. Households discount utility payoffs across periods with the discount factor $\beta \in (0, 1)$; so that the utility function for household i can be represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \{x_t(i) + \omega_t(i)u(c_t(i)) - v(y_t(i))\}. \tag{1}$$

In the spirit of Kocherlakota (2003), a spatial structure is imposed at night with two spatially separated locations: location 1 and location 2. Subsequent to the realization of consumer types, consumers of group 1(2) households move to location 1(2) for consumption of the night output, while producers of group 2(1) households move to location 1(2) for production of the night output. This ensures that the two locations are symmetric in terms of the composition of preference types at night. Moreover, households cannot consume their own output at night as a consequence of this spatial structure.

Entrepreneurs live for two periods and can only participate in the day subperiod. Each day, a generation of young entrepreneurs is born who can consume only in old age and then die in the following day. A young entrepreneur is endowed with an investment opportunity that transforms x_t units of day output at date t to $z_t f(x_t)$ units of day output in date $t + 1$, where $0 < z_t < \infty$ denotes a productivity parameter and $f'' < 0 < f'$, $f'(0) = \infty$, and $f'(\infty) = 0$. The entrepreneur then consumes x_t in date $t + 1$ when he becomes old.

Productivity evolves stochastically over time and follows a Markov process, $Pr[z_{t+1} \leq z^+|\eta_t = \eta] = G(z^+|\eta)$; where G is a cumulative distribution function conditional on information η_t (news) at the beginning of each night. Following Andolfatto and Martin (2013), I assume that news can be classified into two categories: good news and bad news; so that $\eta_t \in \{b, g\}$ and denote $\pi \equiv Pr[\eta_t = b]$. Define $z(\eta) = \int z^+ dG(z^+|\eta)$ and assume that $G(z^+|g) \leq G(z^+|b)$ which implies $z(b) \leq z(g)$. In particular, news η_t received at the beginning of the night is a short-term conditional forecast of next day's productivity. Moreover, good news first-order stochastically dominates bad news. In contrast, $z^e \equiv \pi z(b) + (1 - \pi)z(g)$, is a long-term forecast of productivity that extends to infinite horizons, where $E_t z_{t+1} = z^e$ for all t.

Given the perishability of the day and the night output along with the spatial structure, the

[5]Note that ω_l represents the marginal utility of type l consumers and ω_h represents the marginal utility of type h consumers, after the shock is realized.

resource constraints are as follows

$$\int_{A\cup B} x_t(i)di + x_{t+1} \leq z_t f(x_t), \tag{2}$$

$$\int_A c_t(i)di \leq \int_B y_t(i)di \quad \text{and} \quad \int_B c_t(i)di \leq \int_A y_t(i)di. \tag{3}$$

The planner weights all agents equally and maximizes the aggregate welfare,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t(i) + \omega_t(i)u(c_t(i)) - v(y_t(i)) \right\}, \tag{4}$$

subject to the resource constraints (2) and (3). Note that the symmetry in location means that the night resource constraint (3) can also be expressed as $0.25c_l + 0.25c_h = 0.5y$, with measure 0.25 of type l consumers, measure 0.25 of type h consumers, and measure 0.5 of producers. Because of linear utility in x, the first-best allocation must be consistent with any lottery scheme in $\{x_t(i)\}$ satisfying the expected value $z_t f'(x_t) - x_{t+1}$.[6] Assume, without loss of generality, that for the solution of the first-best allocation, the planner may assign $x_{t+1} = x^*$ for all t; where

$$\beta z^e f'(x^*) = 1. \tag{5}$$

Given the strict concavity of u and strict convexity of v, the first-best allocation is characterized by

$$\begin{aligned} u'(c_l^*) &= \delta u'(c_h^*), \\ u'(c_l^*) &= v'(y^*), \\ c_l^* + c_h^* &= 2y^*. \end{aligned} \tag{6}$$

Note that the first-best allocation is independent of news by construction; see Proposition 1 in Andolfatto and Martin (2013). Moreover, given the preferences and endowment, there are gains from trade between households and entrepreneurs. That is, consumers want to consume the output of the producers, and entrepreneurs want to borrow from households to invest in their investment opportunities. I place further restrictions on this environment that will render trade by credit infeasible, which will provide a role for the bankers to facilitate intertemporal exchange. Bank deposits and interest-bearing money are the possible media of exchange that are essential for trade. In what follows, I assume that both entrepreneurs and households lack commitment. They are also anonymous, which together with lack of commitment rules out enforcement of debt repayment by households. This implies that all trade must be *quid pro quo*. Furthermore, I restrict trade between agents to occur in competitive spot markets.

[6]Given the environment, I mean by first-best allocation is what allocation is best if there is perfect monitoring and agents can commit to future actions.

3 Private economy with banking

I refer to private economy as a competitive equilibrium free of central bank intervention in which bank deposits can be used to facilitate intertemporal exchange. Similar to the entrepreneurs, bankers live for two periods and can only participate during the day. A generation of young bankers is born in the day, but die in the next day after becoming old. Unlike entrepreneurs and households, bankers can commit and are able to enforce repayment of debt at no cost. This allows the banks (owned by bankers) to act as financial intermediaries between the entrepreneurs and households.

As in Chiu et al. (2019), banks want to fund investment projects by issuing liquid deposits which can be used as a means of payment by households in the day market. In this private economy, suppose for now that banks do not receive anything in exchange from households for the deposits issued. Money is thus created *ex nihilo* in the private economy when banks issue deposits to the households. Banks also make loans to entrepreneurs in the form of deposits, which the entrepreneurs use to purchase output x from households for investment. The investment of the entrepreneurs is subject to productivity shocks mentioned earlier. In the night market, households use deposits to trade goods. In the next day, entrepreneurs and households settle their debt by repaying loans and deposits, respectively. After selling some of their investment for deposits to settle bank loans, entrepreneurs can retain the leftover output for their own consumption. Bankers collect loan repayments and redeem deposits held by households. The banking sector is assumed to be perfectly competitive with free entry; so that banks make zero profit. Figure 1 illustrates the timeline of the model.

In what follows, I will characterize a competitive equilibrium in which bank deposits are circulated as exchange media.

3.1 Decision-making of banks and entrepreneurs

I examine the optimization problems faced by banks and entrepreneurs, respectively. Their respective optimization problems will determine the demand and supply of loans and deposits in the equilibrium.

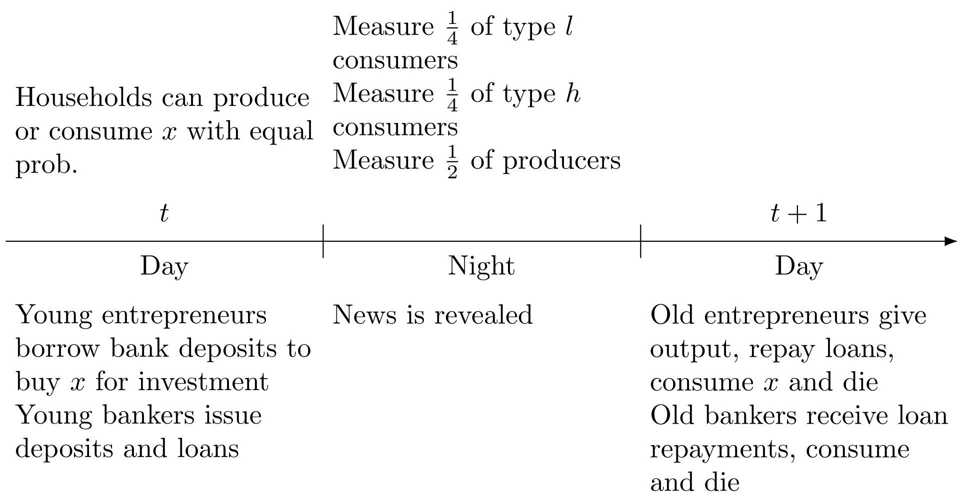

Figure 1: Timeline

3.1.1 Entrepreneurs

Consider the entrepreneurs who take the loan rate $R^L(z)$ as given to maximize consumption, $zf(p)$, in the second period of life (that is, day consumption when old); where p denotes loans. When borrowing from the bank, he/she faces a pledgeability constraint, $p \leq zf(p)$; see Rocheteau et al. (2018). That is, the entrepreneur pledges the entirety of their output from investment to obtain loans from the bank.[7] Since banks can enforce repayments, I am assuming that the debt owed by the entrepreneurs can be recovered fully in the event of default.

Formally, the entrepreneur solves the following maximization problem:

$$\begin{aligned} \max_{p} \quad & \{zf(p) - R^L(z)p\} \\ \text{s.t.} \quad & p \leq zf(p). \end{aligned} \tag{7}$$

There are two cases to consider. First if the pledgeability constraint is slack, then $p = p^*$, where $zf'(p) = R^L(z)$. Second, if the pledgeability constraint binds then $p < p^*$ where $zf'(p) < R^L(z)$. An entrepreneur borrows up to the point where the marginal cost of obtaining the loan equals its marginal benefit. When the marginal cost of obtaining the loan exceeds the marginal benefit, the entrepreneur would not be willing to obtain the loan due to the banks charging a higher $R^L(z)$. The higher $R^L(z)$ stems from the increased risk of default generated by the productivity shock that results in a binding constraint. It follows that the demand for loans decreases with the loan rate. Next, I examine the bank's problem.

[7]There may be restrictions on the amount of output that can be pledged. These restrictions can not only arise from institutions including the legal system, but also from information and commitment frictions. I abstract from such restrictions on pledgeability here.

3.1.2 Banks

Banks issue deposits d to households and invest in loans p offered to the entrepreneurs. Let $\psi_1(z)$ denote the price of deposits at the end of each day. Competitive markets imply that banks take the deposit price $\psi_1(z)$ and the lending rate $R^L(z)$ as given. The pledgeability constraint of the entrepreneurs now translates to a lending constraint for the bankers: $p \leq zf(p)$. Banks also face a balance sheet constraint, $p = d$, where the right-hand side is the liability and the left-hand side is the asset. In this case, the loans represented by p constitute the bank's assets, while the issued bank deposits, denoted by d, serve as the liabilities.. The constraint is the balance sheet identity of the bank. The bank's maximization problem can then be written as

$$\begin{aligned} \max_{p,d} \quad & \left\{R^L(z)p - \psi_1(z)d\right\} \\ \text{s.t.} \quad & p = d, \\ & p \leq zf(p). \end{aligned} \tag{8}$$

Substitute out d using the balance sheet identity and rewrite the bank's maximization problem as

$$\begin{aligned} \max_{p} \quad & \left\{R^L(z)p - \psi_1(z)p\right\} \\ \text{s.t.} \quad & p \leq zf(p). \end{aligned} \tag{9}$$

Once again there are two cases to consider. First, if the lending and pledgeability constraints are slack, then $p = p^*$ where $\psi_1(z) = R^L(z) = zf'(p)$. Second, if the lending and plegeability constraints bind then $p < p^*$ where $\psi_1(z) > R^L(z) > zf'(p)$. The higher risk of loan default—as a result of the productivity shock—implies that the banks will cut back on their lending to firms. This is because the marginal benefit of issuing an extra unit of loan is below its marginal cost. The banks will increase their lending rates, which will make it more expensive for entrepreneurs to borrow, leading to a reduction in loan demand.

3.2 Decision-making of households

I now examine the household maximization problem for the day market, and then describe the producer's problem and the consumer's problem for the night market.

3.2.1 The day market

At the beginning of the day, each household enters with d real deposits priced at $\psi_1(z)$. Let $s \geq 0$ denote the real deposits carried forward into the night market. The day-market budget constraint can then be written as

$$x = \psi_1(z)d - \psi_1(z)s. \tag{10}$$

Let $W(d,z)$ denote the utility value of a household beginning the day with d real deposits when the productivity shock is z; let $V(s,\eta)$ denote the utility value associated with entering the night market with s real deposits conditional on news η. These two value functions must satisfy the following recursive relationship:

$$W(d,z) \equiv \max_{s\geq 0} \left\{\psi_1(z)d - \psi_1(z)s + E_\eta V(s,\eta)\right\}, \tag{11}$$

where V satisfies $\frac{\partial^2 V}{\partial s^2} \leq 0 < \frac{\partial V}{\partial s}$. The demand for real deposits can then be characterized by the first-order condition:

$$\psi_1(z) = E_\eta \frac{\partial V(s,\eta)}{\partial s}. \tag{12}$$

Notice that the optimal choice of s is identical across all households entering the night market. This is because the demand for real deposits is independent of initial deposit holdings d. Furthermore, the envelope condition yields

$$\psi_1(z) = \frac{\partial W(d,z)}{\partial d}. \tag{13}$$

The above condition implies that W is quasilinear in d and given stochastic productivity, the deposit price is time-invariant; that is, $\psi_1(z) = \psi_1^+(z)$.

3.2.2 The night market

At the beginning of the night, households realize whether they are consumers or producers. Consumer preference shock is also realized at the beginning of the night. Then the consumers of type $j \in \{l,h\}$ and the producers in households separate to travel to different locations, as in Xiang (2013). A household makes the consumption and production decisions *ex ante* on behalf of the type j consumer and the producer; instructions of each household are simply carried out by the producers and consumers. News about the entrepreneurs' productivity is also revealed at the beginning of the night.

Let $c_j = c_j^d$ represent the total purchases of output by a type j consumer using bank deposits, and $y_j = y_j^d$ denote the total amount of output produced where deposits are accepted as payments. Because of limited commitment and lack of record keeping, each consumer with realized type j of a household faces a deposit constraint[8]

$$c_j \leq \psi_2(\eta)s. \tag{14}$$

The price of deposits at night is influenced by news. Denote by $\psi_2(\eta)$ the price of deposits at night paid by a household with realized consumer type j. The price is paid to purchase output y_j for consumption c_j. Accordingly, the future deposit balances are given by

$$d_j^+ = \frac{1}{\psi_2(\eta)}\left(\psi_2(\eta)s + y_j - c_j\right).$$

The choice problem for a household with realized consumer type $j \in \{l, h\}$ can be expressed as

$$V_j\left(s,z\right) \equiv \max_{c_j, y_j}\left\{\omega_j u(c_j) - v(y_j) + \beta\,\mathrm{E}\left[W\left(\frac{1}{\psi_2(\eta)}\left(\psi_2(\eta)s + y_j - c_j\right), z^+\right)\middle|\,\eta\right]\right\}. \tag{15}$$

Since a producer has the desire to accumulate deposit balances for future consumption, the deposit constraint $d_j^+ \geq 0$ will not bind. Independent of household types, all producers produce output y; so that the supply of output y at night is characterized by

$$v'(y(\eta)) = \beta\frac{\psi_1(z(\eta))}{\psi_2(\eta)}. \tag{16}$$

The consumption of output c_j at night will depend on whether or not the deposit constraint for type j binds. By applying (13), the desired consumption c_j at night is characterized by

$$\begin{aligned} &\omega_j u'(c_j(\eta)) = \beta\tfrac{\psi_1(z(\eta))}{\psi_2(\eta)} && \text{if } \psi_2(\eta)s \geq c_j(\eta) \\ &c_j(\eta) = \psi_2(\eta)s && \text{otherwise.} \end{aligned} \tag{17}$$

Moreover, the envelope condition in either case is

$$\frac{\partial V_j(s,z)}{\partial s} = \psi_2(\eta)\omega_j u'(c_j(\eta)). \tag{18}$$

[8]The deposit constraint can also be interpreted as a debt-constraint that has been used in many Lagos-Wright type models, but more specifically I am referring to the environments in Andolfatto and Martin (2013) and Andolfatto (2013).

3.3 Equilibrium

Denoting the total supply of deposits by S, the loan demand and loan supply by p^d and p^s, respectively, market-clearing conditions imply

$$\begin{gathered} s = S, \\ 0.25c_l(\eta) + 0.25c_h(\eta) = 0.5y(\eta), \\ p^s = p^d. \end{gathered} \tag{19}$$

Condition (19) states that in each period, the deposit and the loan markets must clear along with the competitive spot markets in the day and night.

To solve for the equilibrium allocation, four cases must be considered.

Case 1 *Both the deposit constraints for type h and type l consumers remain slack, that is,* $\psi_2(\eta)s \geq c_h(\eta)$ *and* $\psi_2(\eta)s \geq c_l(\eta)$.

Case 2 *The deposit constraints for type h consumers remains slack while it binds for type l, that is,* $\psi_2(\eta)s \geq c_h(\eta)$ *and* $\psi_2(\eta)s = c_l(\eta)$.

Case 3 *The deposit constraints for type l consumers remains slack while it binds for type h, that is,* $\psi_2(\eta)s \geq c_l(\eta)$ *and* $\psi_2(\eta)s = c_h(\eta)$.

Case 4 *Both the deposit constraints for type h and type l consumers bind, that is,* $\psi_2(\eta)s = c_h(\eta)$ *and* $\psi_2(\eta)s = c_l(\eta)$.

Considering Case 1, by (17) one obtains

$$u'(c_l(\eta)) = \beta\frac{\psi_1(z(\eta))}{\psi_2(\eta)} = \delta u'(c_h(\eta)). \tag{20}$$

By applying the market clearing conditions, both Case 2 and Case 3 imply

$$\begin{aligned} u'(c_h(\eta)) &= \tfrac{\beta}{\delta}\tfrac{\psi_1(z(\eta))}{\psi_2(\eta)} \quad \text{and} \quad c_l(\eta) = \psi_2(\eta)S, \\ u'(c_l(\eta)) &= \beta\tfrac{\psi_1(z(\eta))}{\psi_2(\eta)} \quad \text{and} \quad c_h(\eta) = \psi_2(\eta)S, \end{aligned} \tag{21}$$

respectively. Applying the market-clearing conditions, Case 4 leads to

$$\psi_2(\eta)S = y(\eta) < y^*. \tag{22}$$

On the other hand, applying the market-clearing conditions for Case 1 results in

$$\psi_2(\eta)S \geq y(\eta) = y^*. \tag{23}$$

For cases in which the deposit constraint for type l consumers does not bind, applying (16) yields

$$u'(c_l(\eta)) = \beta\frac{\psi_1(z(\eta))}{\psi_2(\eta)} = v'(y(\eta)). \tag{24}$$

Similarly, a slack deposit constraint for type h implies $u'(c_h(\eta)) = {}^{v'(y(\eta))}/_{\delta}$. Note that ${}^{\partial V(s,z)}/_{\partial s} = 0.5{}^{\partial V_l(s,z)}/_{\partial s} + 0.5{}^{\partial V_h(s,z)}/_{\partial s}$. Appealing to (13), the following equilibrium restriction must be true,

$$\psi_1(z^e) = \frac{\pi\psi_2(b)}{2}\left[u'(c_l(b)) + \delta u'(c_h(b))\right] + \frac{(1-\pi)\psi_2(g)}{2}\left[u'(c_l(g)) + \delta u'(c_h(g))\right]. \tag{25}$$

Assuming that the deposit constraint for type l consumers is slack, condition (25) may be rewritten as

$$\psi_1(z^e) = \pi\psi_2(b)v'(y(b))A(y(b)) + (1-\pi)\psi_2(g)v'(y(g))A(y(g)), \tag{26}$$

where

$$A(y) \equiv 0.5\left[\frac{\delta u'(y)}{v'(y)} + 1\right]. \tag{27}$$

Notice that $A(y^*) = 1$ and $A'(y) < 0$.

Next, from the respective maximization problems of the entrepreneurs and banks, one can derive the deposit-price function

$$\psi_2(\eta) = \beta\frac{z(\eta)f'(p)}{v'(y(\eta))} = \beta\frac{R^L(z^e)}{v'(y(\eta))}, \tag{28}$$

by assuming that the plegeability and lending constraints of the entrepreneurs and banks, respectively, are slack. The equilibrium allocation $(c_l(\eta), c_h(\eta), y(\eta))$ at night in which only bank deposits is used as a medium of exchange is characterized by the conditions (22), (23), (26) and (28). Next, I consider cases in which news may or may not be of importance in this private economy.

3.3.1 Equilibrium with no news

In this section, I examine the competitive equilibrium where news is of no importance. This means $z(\eta) = z^e$ for $\eta \in \{b, g\}$ with the implication that $y(\eta) = y$ and $\psi_2(\eta) = \psi_2$.

I now seek to solve for ψ_1. Notice that with no news,

$$\psi_2 = \beta \frac{z^e f'(p)}{v'(y)} = \beta \frac{R^L}{v'(y)}. \tag{29}$$

Combining (26) with (28) yields the following expression for the deposit day-price

$$\psi_1 = \beta z^e f'(p) A(y) = \beta R^L A(y) > 0. \tag{30}$$

Condition (30) states the rate of return on bank deposits must compensate for discounting across time by the loan rate or by the expected marginal product of the day output. Following Andolfatto and Martin (2013), I refer to this condition as the "fundamental" price of deposits, as this reflects the average price relative to extreme price fluctuations. Given the strict concavity of u, from an *ex ante* perspective, society prefers average prices to extremes to smooth out consumption over time. Note that the deposit price also depends on the preference parameter δ. We have the following proposition.

Proposition 1 *ψ_1 is strictly increasing in δ.*

Proof. Rewriting the deposit day-price by using the definition of $A(y)$ from (27) gives us

$$\psi_1 = \frac{\beta z^e f'(p) \left[\delta u'(y) + v'(y)\right]}{2v'(y)} = \frac{\beta R^L \left[\delta u'(y) + v'(y)\right]}{2v'(y)} > 0.$$

Differentiating the expression above w.r.t. δ gives rise to

$$\frac{\partial \psi_1}{\partial \delta} = \frac{\beta z^e f'(p) u'(y)}{2v'(y)} = \frac{\beta R^L u'(y)}{2v'(y)} > 0.$$

■

The interpretation of Proposition 1 is as follows. Since all consumption in the night market must be purchased by using deposits, the consumer preference shock can be interpreted as a liquidity shock; where δ measures the magnitude of this shock. An increase in the magnitude of this liquidity shock is reflected in a higher deposit price during the day in the form of a liquidity premium.[9] This is due to the high demand from type h consumers for the night output as δ gets

[9] If an asset is used as a medium of exchange then the asset will be traded at a premium relative to other illiquid assets. The fact that financial assets are valued for their liquidity when they are used as exchange media has been

larger and also because of the usefulness of bank deposits as a means of payment. Moreover, given $A(y^*) = 1$ and using the restriction $\beta z^e f'(p^*) = 1$ (from the solution in the planner's problem in (5)) implies $\psi_1^* = 1$, $R^{L^*} > 0$ and $p = p^*$.

Next, I verify the conditions under which the deposit constraint for either type of consumers will not bind, that is, $\psi_2 S \geq y^*$. First, suppose that both the pledgeability constraint and the lending constraint from the entrepreneur's and the bank's optimization problems are slack, that is, $p \leq zf(p)$. This implies $p = p^*$. Using (29), condition $\psi_2 S \geq y^*$ can be expressed in terms of parameters,

$$\beta \geq \frac{y^* v'(y^*)}{S z^e f'(p^*)} = \frac{y^* v'(y^*)}{S R^{L^*}}. \tag{31}$$

Defining

$$\beta^*(z^e, R^L) = \frac{y^* v'(y^*)}{S z^e f'(p^*)} = \frac{y^* v'(y^*)}{S R^{L^*}} \tag{32}$$

as the equilibrium object corresponding to the efficient level of production y^*, we get the following result.

Proposition 2 *If $\beta \in \left[\beta^*(z^e, R^L), 1\right)$ and $p \leq zf(p)$, then $p = p^*$ and where R^{L^*} corresponds to $zf'(p) = R^L(z)$. A competitive equilibrium corresponds to the efficient allocation y^*. Moreover, $\beta^*(z^e, R^L)$ is independent of δ.*

Liquidity shock has no influence on the parameters for which the efficient allocation can be implemented. Observe that $\beta^*(z^e, R^L)$ is strictly decreasing in z^e and R^L. The higher the risk of loan default is, the lower is the bank lending to firms; so that $R^L < \psi_1(z^e)$. This is up to the point where the pledgeability constraint and the lending constraint for both the firms and banks may bind, that is, $p = zf(p)$. The efficient allocation is only implementable for patient economies—that is, for economies with sufficiently high β— and up to the point on which the set of economies can be expanded. Beyond this point, an efficient allocation is no longer feasible.

A liquidity shortage arises for impatient economies—that is, for the case when $\beta \in \left(0, \beta^*(z^e, R^L)\right]$ and $p = zf(p)$—in the sense of Caballero (2006); see also Proposition 2 in Andolfatto and Martin (2013) for an analogous result. Since entrepreneurs pledge their future output as collateral to the banks, limited commitment means that liquidity is in short supply. Owing to a lack of commitment from the entrepreneurs, there is a liquidity shortage when the pledgeable future output is subject to binding constraints as a result of technological uncertainty. Hence, deposits are in short supply, creating a liquidity shortage. This makes the deposit

highlighted in Lagos (2010).

constraints for both type l and type h consumers to bind tightly; so that $\psi_2(\eta)S = y(\eta) < y^*$.

When deposit constraints bind for both consumer types, combined with the restrictive lending constraints of the banks and the pledgeability constraints of the entrepreneurs, the deposit-price function (30) indicates an overvaluation of the bank deposit compared to its fundamental value. Though household members wish to borrow money from banks overnight, restrictions imposed by both the banks and entrepreneurs from the previous day hinder this possibility. Entrepreneurs, eager to secure loans from banks, find themselves constrained, leading to banks' reluctance to lend. Since $A(y) > 1$, the expected rate of return on deposit is

$$\frac{\psi_1}{\beta} > \psi_1 > R^L > z^e f'(p) > 0,$$

which suggests that the effect of a liquidity shortage is to confer a liquidity premium on the price of deposit when bank deposit is used as a medium of exchange. The implication of this liquidity premium is that the deposits will earn a lower expected rate of return, as originally highlighted in Lagos and Rocheteau (2008).

3.3.2 Equilibrium with news

I now consider the case when news is of importance. This means that $z(b) < z^e < z(g)$. I present the results that are similar to Andolfatto and Martin (2013), but when the consumer deposit constraints for both types, along with the banks and the entrepreneurs respective constraints are considered.

Lemma 1 *The deposit constraints for both type l and type h consumers cannot remain slack in both news states along with slack lending and plegeability constraints for the banks and the entrepreneurs, respectively.*

Proof. Suppose that Case 1 holds in both news states, that is, the deposit constraints for both type l and type h are slack in both news states. Also, assume that the lending and the pledgeability constraints of the banks and the entrepreneurs, respectively, are slack. Then $y(b) = y(g) = y^*$ and $p = p^*$. In this case, $A(y^*) = 1$ and so by condition (30),

$$\psi_1 = \beta z^e f'(p^*) = \beta R^{L^*}.$$

Moreover, condition (29) imply

$$\psi_2(\eta) = \beta \frac{z(\eta) f'(p^*)}{v'(y^*)}.$$

Using (23), $\beta \frac{z(\eta)f'(p^*)}{v'(y^*)} \geq {}^{y^*}\!/s$ for $\eta \in \{b, g\}$. This implies $\beta z(b) f'(p^*) \geq {}^{y^*v'(y^*)}\!/s$. Since $z(b) < z^e < z(g)$, it follows that

$$\psi_1 = \beta z^e f'(p^*) > \beta z(b) f'(p^*) \geq {}^{y^*v'(y^*)}\!/s = S\beta R^{L^*} = S\psi_1,$$

by using condition (32). But this is a contradiction; as $S > 0$. ■

Lemma 2 *The deposit constraints for both type l and type h consumers cannot bind tightly in both news states along with binding plegability and lending constraints.*

Proof. Assume that Case 4 holds in both news states, that is, the deposit constraints for both type l and type h bind tightly in both news states. Also, assume that the constraints for the banks and the entrepreneurs bind. Then (26) and (28) imply

$$\psi_1(z^e) = \pi\beta z(b) f'(p) A(y(b)) + (1 - \pi)\beta z(g) f'(p) A(y(g)).$$

If the debt constraints for all the agents bind, then (22) implies $y(\eta) < y^*$ for $\eta \in \{b, g\}$. This means that $A(y(\eta)) > 1$ for $\eta \in \{b, g\}$ and $p < p^*$ given binding pledgeability and lending constraints. Then by the inequality in (31),

$$\psi_1 > \beta z^e f'(p) = \beta R^L > \frac{y^* v'(y^*)}{S}.$$

Moreover, condition (22) implies $\psi_2(\eta)S = y(\eta) < y^*$ for $\eta \in \{b, g\}$. Then (28) implies

$$y(b)v'(y(b)) = \beta z(b) f'(p),$$

$$y(g)v'(y(g)) = \beta z(g) f'(p).$$

Since $z^e = \pi z(b) + (1 - \pi)z(g)$, the two equalities above imply that

$$\pi y_2(b)v'(y(b)) + (1 - \pi)y_2(g)v'(y(g)) = \beta z^e f'(p) = \beta R^L.$$

Therefore,

$$\pi y_2(b)v'(y(b)) + (1 - \pi)y_2(g)v'(y(g)) > \frac{y^* v'(y^*)}{S}.$$

However, this is impossible; as $y'v'(y)$ is strictly increasing in y, and as $y(\eta) < y^*$. ■

Lemma 3 *The deposit constraints for both type l and type h consumers cannot bind tightly in the good-news state along with binding plegability and lending constraints. The deposit* *constraints for both types cannot remain slack in the bad-news state along with slack plegeability and lending constraints.*

Proof. Suppose that the deposit constraints for both types of consumers bind tightly in the good-news state and remain slack in the bad-news state. Then (22) and (23) imply $\psi_2(b)S > y^*$ and $\psi_2(g)S = y(g) < y^*$. Also, by condition (28)

$$\psi_2(b)v'(y^*) = \beta z(b)f'(p^*) = \beta R^{L^*},$$

$$\frac{y(g)v'(y(g))}{S} = \beta z(g)f'(p^*) = \beta R^{L^*}.$$

Observe that these two latter restrictions together with $z(g) > z(b)$ imply that

$$\frac{y(g)v'(y(g))}{S} > \psi_2(b)v'(y^*) > \frac{y^*v'(y^*)}{S},$$

which is a contradiction; as $yv'(y)$ is strictly increasing in y and as $y(g) < y^*$. ■

The three lemmas above effectively rule out Case 2 and 3. The remaining possibility is presented below, which corresponds to Proposition 3 in Andolfatto and Martin (2013).

Proposition 3 *If $z(b) < z^e < z(g)$ and $\beta = \beta^*(z^e, R^L)$, then the consumer deposit constraints for both type l and type h bind tightly in the bad-news state when $p = zf(p)$ and $p < p^*$, along with binding lending and plegeability constraints for the banks and the entrepreneurs, respectively. The deposit constraints for both types become slack in the good-news state when $p \leq zf(p)$ and $p = p^*$, along with slack lending and plegeability constraints.*

Proposition 3 fixes a pair $\beta(z^e, R^L)$ so that the competitive equilibrium barely implements the efficient allocation when there is no news. Following Andolfatto and Martin (2013), I perform a mean-preserving spread over the short-run conditional forecast of future productivity. Specifically, I keep z^e fixed and increase the variance of the short-run forecast around this mean. The analysis follows from their paper, namely that good news slackens a weakly binding constraint while bad news induces the deposit constraints of both type l and type h consumers to bind tightly. Despite potential fluctuations in deposit prices during short-term news events related to entrepreneurs' productivity, bank deposits can still function as exchange media, as long as efficiency can be maintained. The implementation of an efficient allocation is possible if consumers of both types are not debt-constrained in either news state, which will correspond to the case when the entrepreneurs' pledgeability constraint and the bank's lending constraint are not binding.

Proposition 3 implies $\psi_2(b)S = y(b) < \psi_2(g)S = y(g) = y^*$. Combining (26) with (28), we have the price of deposits in the day

$$\psi_1 = \beta R^L \left[\pi A(y(b)) + (1-\pi)A(y(g))\right]$$

Note that the above equation reduces to (30) when $y(b) = y^*$. Since $\psi_1 > 0$ and $\psi_1 = \beta R^L < 1$ when $y(b) = y(g) = y^*$, we have the following restriction that characterizes the equilibrium

$$\frac{\psi_1}{R^L} = \beta \left[\pi A(y(b)) + (1-\pi)A(y(g))\right]. \tag{33}$$

The implication of (33) is that information itself will carry a premium in the day-deposit price. Due to uncertainty, banks will raise their deposit rate, so that $\psi_1 > \psi_1^* > R^L$ as opposed to the no-news case. Here, information itself carries a premium in how the deposit prices will be set by the banks. The high deposit price means that bank lending in the news economy may be be suboptimal, that is, $p^* > p$.

As for the equibrium price of deposits at night, once again recalling condition (28)

$$\psi_2(b) = \beta \frac{z(b) f'(p)}{v'(y(b))} = \beta \frac{R^L}{v'(y(b))},$$

and

$$\psi_2(g) = \beta \frac{z(g) f'(p^*)}{v'(y^*)} = \beta \frac{R^{L^*}}{v'(y^*)}.$$

We have $\psi_2(g) > \psi_2(b)$; an implication of Proposition 3 due to the deposit constraints for both types become slack in the good-news state and binding in the bad-news state, and also when v is linear (a special case).

Next, I examine how central bank intervention with interest-bearing money can coexist with bank deposits. More specifically, I explore whether interest-bearing money can help overcome the liquidity shortage in the news economy when $\beta \in \left(0, \beta^*(z^e, R^L)\right]$.[10]

[10]The implicit assumption made in this context is that a nondisclosure of news is infeasible, that is, society does not have the power to hide bad news from the individuals. Hiding bad news and revealing good news is only time-consistent for sufficiently patient economies. This is the main motivation for government or central bank intervention for this class of models; see Andolfatto and Martin (2013) for more details.

4 Bank deposits and interest-bearing money

The central bank intervenes at the night market after the realization of agents' types. I assume that $\omega_t(i)$ is publicly observable upon realization. This assumption implies that the central bank can operate on a type-contingent transfer policy by observing household types. The type-contingent transfer policy is introduced along the lines of Andolfatto (2011). Lump-sum taxation in the day market is ruled out and assume money as a divisible object. Let $\{c, p\}$ denote the household types, which are categorized into consumers and producers. The central bank's policy rule is to make lump-sum transfers of money $T_j^\iota \geq 0$ at each night after observing these household types, where $\iota \in \{c, p\}$. The central bank also pays a positive nominal interest rate $R^M \geq 1$ on money balances. Banks are required by the central bank to hold a fraction ρ of their deposits as currency reserves.

Banks now issue deposits to households in exchange for interest-bearing money which can be retained as bank reserves. Banks still make loans to entrepreneurs in the form of deposits. Loans and deposits are settled in the following day. Entrepreneurs use either deposits or interest-bearing money to purchase the day good from households. Households use a combination of bank deposits and interest-bearing money to trade goods in the night market. The process is the same as described earlier other than interest-bearing money now coexisting with deposits.

Denote by (ϕ_1, ϕ_2) the value of money in the day and night markets, respectively. Let M denote the total stock of money at the beginning of the day; with M^+ denoting the "next" period's money supply. Assume that this stock evolves at the constant gross rate $\mu \geq 1$, so that $M^+ = \mu M$. Since the central bank makes lump-sum transfers and also pays a nominal interest rate, the central bank budget constraint must satisfy $\left(R^M - 1\right) M = M^+ - M + 0.25T_l^c + 0.25T_h^c + 0.5T^p$, where $(R^M - 1)$ is the central bank's aggregate interest obligation. Suppose $T_l^c = T^p = 0$. Then, $T_h^c = 4\left(\frac{R^M}{\mu} - 1\right) M^+$, or in real terms can be expressed by

$$\tau_h^c = 4\left(\frac{R^M}{\mu} - 1\right)\phi_1 M^+, \tag{34}$$

where $\tau_j^c \equiv \phi_1 T_j^c$. Note that linearity restricts the transfes to be proportional.

4.1 Decision-making of banks and entrepreneurs with interest-bearing money

Note that the optimization problem of the entrepreneurs stays the same as before. I now examine the optimization problem of banks when they have the option of investing in government-issued interest-bearing reserves.

4.1.1 Banks

Banks now issue deposits d to households and invest in loans p and interest-bearing money m_1 issued by the central bank, where $m_1 \geq 0$ is the nominal money balances during the day. Banks acquire the real quantity of outside interest-bearing money, $a \equiv \phi_1 m_1$, and earn interest R^M. Both the deposit market and the loan market are competitive as before. Banks also face a reserve requirement. At the end of each date, the bank's beginning-of-the-day real money balances, a, must be at least ρ fraction of the total deposits, that is, $\rho d \leq a$, where ρ is a policy parameter set by the central bank. The bank solves the following maximization problem:

$$\begin{aligned} \max_{p,d,a} \quad & \left\{ R^L(z)p + R^M a - \psi_1(z)d \right\} \\ \text{s.t.} \quad & p + a = d, \\ & \rho d \leq a, \\ & p \leq zf(p). \end{aligned} \tag{35}$$

Once again, substitute out d using the balance sheet identity and rewrite the bank's maximization problem as

$$\begin{aligned} \max_{p,d} \quad & \left\{ \left(R^L(z) - R^M \right) p + \left(R^M - \psi_1(z) \right) d \right\} \\ \text{s.t.} \quad & p \leq (1-\rho)d, \\ & p \leq zf(p). \end{aligned} \tag{36}$$

There are several cases to consider. For the first case, suppose that the reserve requirement, the lending and the pledgeability constraints are all slack. Then $p = p^*$ when $zf'(p) = R^L(z) = R^M = \psi_1(z)$. When the marginal benefit of investing in loans is equal to the marginal benefit of investing in interest-bearing money, the bank is indifferent between investing in loans and investing in interest-bearing money. This is because both interest-bearing reserves and loans have the same rate of return. For the second case, suppose that the reserve requirement is slack but the lending and plegeability constraints bind. Then $zf'(p) < R^L(z) < R^M = \psi_1(z)$ and $p < p^*$. Interest-bearing reserves have a higher return than loans; so that banks will reduce their lending and hold more cash reserves (earns interest) due to a higher risk of loan default from the entrepreneurs. In this way, banks invest in interest-bearing reserves as insurance against the limited commitment friction of the entrepreneurs. If the reserve requirement binds along with binding plegeability and lending constraints, then $zf'(p) < R^L(z) < R^M < \psi_1(z)$. In words: if the reserve requirement binds, the bank will need to charge a higher deposit rate than the interest it earns from reserves given the productivity shock z, which the bank is exposed to from the entrepreneur's collateral.

4.2 Decision-making of households

In this section, I examine the household maximization problem for the day market, and then describe the producer's problem and the consumer's problem for the night market when interest-bearing money and bank deposits may coexist as exchange media.

4.2.1 The day market

A household enters the day with m_1 nominal money balance. Let m_2 denote the nominal money balance taken by this household into the night market. Recall that we already defined the real money balance at the beginning of the day as $a \equiv \phi_1 m_1$ in the bank's maximization problem. Define the real money balance carried forward into the night $q \equiv \phi_1 m_2$. The day budget constraint of a household is now given by

$$x = \psi_1(z)d - \psi_1(z)s + R^M a - q. \tag{37}$$

Analogous to (11), the choice problem in the day is

$$W(d, a, z) \equiv \max_{s \geq 0, q \geq 0} \left\{ \psi_1(z)d - \psi_1(z)s + R^M a - q + E_\eta V(s, q, \eta) \right\}. \tag{38}$$

The demand for real deposits and real money, respectively, must satisfy

$$\psi_1(z) = E_\eta \frac{\partial V(s, q, \eta)}{\partial s}, \tag{39}$$

$$1 = E_\eta \frac{\partial V(s, q, \eta)}{\partial q}. \tag{40}$$

The envelope conditions are

$$\psi_1(z) = \frac{\partial W(d, a, z)}{\partial d}, \tag{41}$$

$$R^M = \frac{\partial W(d, a, z)}{\partial a}. \tag{42}$$

Note that the conditions above imply that both $\psi_1(z)$ and ϕ_1 are invariant over time in a stationary equilibrium.

4.2.2 The night market

Households take portfolio (s, q) into the night market, when the news is η. Consumers and producers separate and move to their respective locations. A type j consumer receives a lump-sum transfer of money T_j^c, and travels to another location with real money balances $q + \tau_j^c$. Denote by $c_j = c_j^d + c_j^m$ the total real purchases of output of a type j consumer with deposits and cash, where c_j^d is the output purchased by using deposits and c_j^m is the output purchased by using interest-bearing fiat money. Also denote by $y_j = y_j^d + y_j^m$ the amount of the output produced where a combination of deposits and money is accepted for purchase, with y_j^d as the amount of output that can be purchased by using deposits and y_j^m as the amount of output that can be purchased by using money. In addition to the deposit constraint (14), each consumer type j now faces a cash constraint

$$c_j^m \leq \frac{\phi_2(\eta)}{\phi_1}\left(q + \tau_j^c\right), \tag{43}$$

respectively. The combined deposit and cash constraints can be viewed as a single consumer debt-constraint, defined as

$$c_j \leq \psi_2(\eta)s + \frac{\phi_2(\eta)}{\phi_1}\left(q + \tau_j^c\right).$$

The nominal money balances brought forward by a household into the next day are $m_1^+(j) = m_2 + T_j^c + {}^1\!/_{\phi_2(\eta)}\left(y_j^m - c_j^m\right)$, which can be expressed in real terms,[11]

$$a_j^+ = \frac{\phi_1^+}{\phi_1}\left(q + \tau_j^c + \frac{\phi_1}{\phi_2(\eta)}\left(y_j^m - c_j^m\right)\right).$$

The choice problem for a household with realized consumer type $j \in \{l, h\}$ can be stated as

$$\begin{aligned} V_j(s, q, z) \equiv \max_{c_j^d, c_j^m, y_j^d, y_j^m} \Bigg\{ & \omega_j u(c_j) - v(y_j) \\ & + \beta \mathrm{E}\Bigg[W\Bigg(\frac{1}{\psi_2(\eta)}(\psi_2(\eta)s + y_j^d - c_j^d), \\ & \frac{\phi_1^+}{\phi_1}(q + \tau_j^c + \frac{\phi_1}{\phi_2(\eta)}\left(y_j^m - c_j^m\right)), z^+\Bigg) \Bigg| \eta \Bigg]\Bigg\}. \end{aligned} \tag{44}$$

[11]Since $a_j^+ \equiv \phi_1^+ m_1^+(j)$, multiplying by ϕ_1^+ gives $a_j^+ = \phi_1^+ m_2 + \phi_1^+ T_j^c + {}^{\phi_1^+}\!/_{\phi_2(\eta)}\left(y_j^m - c_j^m\right)$. Again, multiplying by ϕ_1, the evolution of real money balances may be stated, alternatively, as $a_j^+ = {}^{\phi_1^+}\!/_{\phi_1} q + {}^{\phi_1^+}\!/_{\phi_1} \tau_j^c + {}^{\phi_1^+}\!/_{\phi_2(\eta)}\left(y_j^m - c_j^m\right)$.

I want to restrict attention to equilibria in which bank deposits and interest-bearing money coexist. For both of these two assets to be accepted as payment, their expected rate of return from the night to the next day (conditional on news η) must be equal. That is, the following no-arbitrage condition must hold:

$$\frac{\psi_1(z(\eta))}{\psi_2(\eta)} = \frac{R^M \phi_1^+}{\phi_2(\eta)}. \tag{45}$$

Following similar steps as before, the total supply of output y at night is characterized by

$$v'(y(\eta)) = \frac{\beta R^M \phi_1^+}{\phi_2(\eta)}. \tag{46}$$

Applying (39) and (40), the total consumption of output c_j at night is characterized by

$$\begin{aligned} &\omega_j u'(c_j(\eta)) = \tfrac{\beta R^M \phi_1^+}{\phi_2(\eta)} && \text{if } \psi_2(\eta)s + \tfrac{\phi_2(\eta)}{\phi_1}\left(q + \tau_j^c\right) \geq c_j(\eta) \\ &c_j(\eta) = \psi_2(\eta)s + \tfrac{\phi_2(\eta)}{\phi_1}\left(q + \tau_j^c\right) && \text{otherwise.} \end{aligned} \tag{47}$$

In either case, the envelope conditions are

$$\frac{\partial V_j(s,q,z)}{\partial s} = \psi_2(\eta)\omega_j u'(c_j(\eta)), \tag{48}$$

$$\frac{\partial V_j(s,q,z)}{\partial q} = \frac{\phi_2(\eta)}{\phi_1}\omega_j u'(c_j(\eta)). \tag{49}$$

4.3 Equilibrium

Denoting the supply of money by Q and defining $\phi_1 M^+ \equiv Q$, the market-clearing conditions in a monetary equilibrium now imply

$$\begin{aligned} s &= S, \\ q &= Q, \\ 0.25c_l(\eta) + 0.25c_h(\eta) &= 0.5y(\eta), \\ p^s &= p^d. \end{aligned} \tag{50}$$

Note that (34) and the money-market clearing condition, $q = Q$, together can be used to express the lump-sum transfers received by type h consumers as shown below

$$\tau_h^c = 4\left(\frac{R^M}{\mu} - 1\right)Q. \tag{51}$$

In a stationary equilibrium all the real variables are constant over time, so that $S = S^+$ and $Q = Q^+$. It follows that $\phi_1^+/\phi_1 = 1/\mu$.

As before, the four cases will still apply. That is, with market-clearing, conditions (20), (21), (22), and (23) can be restated as

$$u'(c_l(\eta)) = \frac{\beta R^M \phi_1}{\phi_2(\eta)} = \delta u'(c_h(\eta)), \tag{52}$$

$$\begin{aligned} u'(c_h(\eta)) &= \tfrac{\beta R^M \phi_1}{\phi_2(\eta)} \quad \text{and} \qquad c_l(\eta) = \psi_2(\eta)S + \tfrac{\phi_2(\eta)}{\phi_1}Q, \\ u'(c_l(\eta)) &= \tfrac{\beta R^M \phi_1}{\phi_2(\eta)} \quad \text{and} \quad c_h(\eta) = \psi_2(\eta)S + \tfrac{\phi_2(\eta)}{\phi_1}\left(Q + \tau_h^c\right), \end{aligned} \tag{53}$$

$$\psi_2(\eta)S + \frac{\phi_2(\eta)}{\phi_1}Q + 0.5\tau_h^c = y(\eta) < y^*, \tag{54}$$

$$\psi_2(\eta)S + \frac{\phi_2(\eta)}{\phi_1}Q + 0.5\tau_h^c \geq y(\eta) = y^*. \tag{55}$$

Once again, invoking the envelope conditions allows us to get an equivalent condition to (26) that may characterize the monetary equilibrium. Following similar steps, condition (40) may be stated as

$$\phi_1 = \frac{\pi\phi_2(b)}{2}\left[u'(c_l(b)) + \delta u'(c_h(b))\right] + \frac{(1-\pi)\phi_2(g)}{2}\left[u'(c_l(g)) + \delta u'(c_h(g))\right], \tag{56}$$

which by assuming that the type l deposit constraint is slack can be rewritten as

$$\phi_1 = \pi\phi_2(b)v'(y(b))A(y(b)) + (1-\pi)\phi_2(g)v'(y(g))A(y(g)). \tag{57}$$

Note that deposit price is still characterized by condition (28). In fact, in a monetary economy, the deposit price can also be expressed as a function of nominal interest rate,

$$\psi_2(\eta) = \frac{\beta R^M}{v'(y(\eta))}, \tag{58}$$

when considering slack reserve requirement and lending constraints.

Condition (58) shows a channel through which a policy of paying interest on money can influence asset prices. We want to derive an analogous condition for the value of money at night. From condition (46) describing the optimal behavior of the household, we can derive the expression

$$\phi_2(\eta) = \frac{\beta R^M \phi_1}{\mu v'(y(\eta))}. \tag{59}$$

Conditions (54), (55), (57) and (59), together with conditions (22), (23), (26) and (28) derived earlier, characterize the competitive equilibrium allocation at night in which both bank deposits and interest-bearing money are valued. Furthermore, after some manipulation, condition (56) may be rewritten as

$$\frac{\mu}{R^M} = \beta\left[\pi A(y(b)) + (1-\pi)A(y(g))\right]. \tag{60}$$

Note that a stationary monetary equilbrium will concurrently require the equilibrium price of deposit $0 < \psi_1(z) < \infty$ to satisfy the restriction in (33). This gives rise to the following proposition.

Proposition 4 *i) In a news economy with bank deposits and interest-bearing money, the type-contingent transfer policy $R^{M^*} = \beta^{-1} > \mu^* = 1$, $\tau_h^{c*} = 4\left(\frac{1-\beta}{\beta}\right)Q > 0$, and $\tau_l^{c*} = \tau^{p*} = 0$ implements the efficient allocation y^*. The lending market may be suboptimal with $p < p^*$.*
ii) In a no-news economy, there does not exist a monetary equilibrium when $R^{M^*} = \mu^* \geq 1$ and $p = p^*$.

Proof. Since $A(y^*) = 1$, condition (60) is satisfied with $R^{M^*} = \beta^{-1} > \mu^* = 1$. We also need to check the condition that can guarantee positive money balances along with $\psi_1(z) > 0$ by also satisfying condition (33). Substituting τ_h^{c*} into (55) means that we require

$$\phi_1 \geq \frac{\mu^*[y^* v'(y^*) - \beta z(b) f'(p)S] - 2(R^{M^*} - \mu^*)v'(y^*)Q}{\beta R^{M^*} Q}$$

$$\text{or, } \phi_1 \geq \frac{\mu^*[y^* v'(y^*) - \beta R^L S] - 2(R^{M^*} - \mu^*)v'(y^*)Q}{\beta R^{M^*} Q},$$

where $R^{L^*} < R^L$ and $p < p^*$ by assuming that the pledgeability constraint, the lending constraint and the reserve requirement are all binding. This is because from (32), $y^* v'(y^*) = \beta z^e f'(p^*)S > \beta z(b) f'(p) = \beta R^L S$. It follows that any value $\phi_1 < \infty$ satisfying the above inequalities is a competitive monetary equilibrium that will guarantee positive money balances. To see this, note that we can rearrange the above inequalities to write down

$$q = \frac{\mu^*[y^*v'(y^*) - \beta z(b)f'(p)S]}{\beta R^{M^*}\phi_1 + (R^{M^*} - \mu^*)2v'(y^*)} > 0.$$

If $y^*v'(y^*) = \beta z^e f'(p^*)S = \beta R^{L^*}S$ then $q = 0$, which implies that interest-bearing money cannot coexist with bank deposits in the no-news case. ■

Note that with interest-bearing money, the implementation of an efficient allocation is independent of parameters β and z^e. Since I have assumed that a lump-sum tax instrument is not available to the central bank, the standard Friedman rule of setting $(R^M, \mu) = (1, \beta)$ is not feasible. Hence, deflation is not optimal. Taxes cannot be collected by the central bank to finance a deflationary policy. Instead, running an inflationary policy can help overcome a liquidity shortage with a positive nominal interest rate on money. This is because β is strictly decreasing in R^M; so that a higher nominal interest rate and positive inflation expands the set of economies for which the efficient allocation is achievable. This result is in contrast to Andolfatto and Martin (2013), where a stationary monetary equilibrium does not coexist with another asset when there is a constant supply of fiat money, namely, $\mu \geq 1$ (see Proposition 5 in Andolfatto and Martin (2013)). Here, paying nominal interest rates and positive inflation rate makes up for the lack of power to lump-sum tax, which in turn may prevent the liquidity shortage that arises as a result of the technological uncertainty faced by the entrepreneurs with limited commitment.

The central bank possesses the capacity to generate assets from the day good x through the issuance of an interest-bearing debt instrument. By investing in interest-bearing reserves, banks can insure against the volatility associated with technological risks in entrepreneurial ventures. However, the lending market may still be suboptimal due to the uncertain nature of information and also due to the short supply of commitment from the entrepreneurs. The crucial assumption used here is that the central bank can observe household preferences, which allows for the type-contingent transfers conditional on household types. The main goal of these type-contingent transfers is to redistribute the purchasing power of households in a manner that is socially desirable.

Not surprisingly, money introduced in this manner cannot coexist with bank deposits in a no-news economy. This is because in the no-news case, bank deposits operating as the sole medium of exchange can achieve the first-best solution. Since there is no benefit from introducing an asset that is dominated in the rate of return, interest-bearing money is not valued and is therefore redundant. After all, if the economy is functioning to its best capacity with private money then why would there be any reason for the central bank or the government to intervene? Outside money in this case is not essential, as money creation does not improve ex ante welfare relative to what can be achieved with private money.[12] The idea here is quite similar to Proposition 3 in Andolfatto and Martin (2009).

One advantage of including interest-bearing money in this manner is that it does not require us to artificially impose a cash-in-advance constraint on bank deposits to essentially evade the price volatility in deposits, as was highlighted in Andolfatto and Martin (2013). That is, imposing the constraint of $s = 0$, so that individuals can only use money to settle their debt in the night market does not confer any substantial welfare gains. Even though money is affected by news, the nominal interest rate R^M is an additional policy tool (apart from the money growth rate μ) that makes money less sensitive to news. This is considering that the economy experiences the adverse effects of excessive price sensitivity of bank deposits due to information frictions, and particularly when commitment in financial markets is limited.

To see how an additional policy tool R^M can be beneficial even in the deposit market, referring to (58) results in the following proposition.

Proposition 5 *In a news economy, $\psi_2(\eta)$ is increasing in R^M.*

The proposition above asserts that interest-bearing money can provide an additional policy tool to alleviate depressed asset prices. Since $\psi_2(b) < \psi_2(g) = \beta R^{M^*}/v'(y^*)$, raising the interest rate on bank reserves during tougher economic times may provide relief and help the economy recover by relaxing the debt constraints of the banks and consumers. With the intended policy design of promoting financial intermediation, undervalued real deposit prices can reach their long-run fundamental value. In practical terms, this could also be a motivation for introducing interest-bearing CBDC, although this paper only studies a weak-form of CBDC.[13]

To see if there is some empirical evidence to support this proposition, I obtain time series on interest rate on excess reserves (IOER) and interest rates on transaction deposits. The IOER data were obtained from FRED, and the rate on transaction deposits was sourced from Wharton Research Data Services (WRDS) using the Stata code by Chiu et al. (2019).[14] Figure 2 lends some credence to Proposition 5, namely, that there is a positive association between the deposit rate and the IOER.

Is it always reasonable to assume that the central bank can observe household preferences? Probably not. The next section explores this limitation.

[12] See Wallace (2014) for an exposition on the sufficient conditions that can guarantee the essentiality of money.

[13] A weak of form of CBDC because money in this model does not solve the problem of private information. The technology here is not superior to prevent individuals from hiding their money balances if they have private information about their types.

[14] Chiu et al. (2019) obtained the data on interest rates on transaction deposits from the WRDS by using the SAS codes by Drechsler et al. (2017). They obtained the rates on transaction deposits by first dividing interest expenses on transaction accounts (item code: RAID4508) by total transaction deposits (RCON2215) to obtain the quarterly rates for each bank. They then obtained a quarterly industry average by taking a weighted average across banks by taking into account their transaction deposits. See their paper for more details on the data methodology.

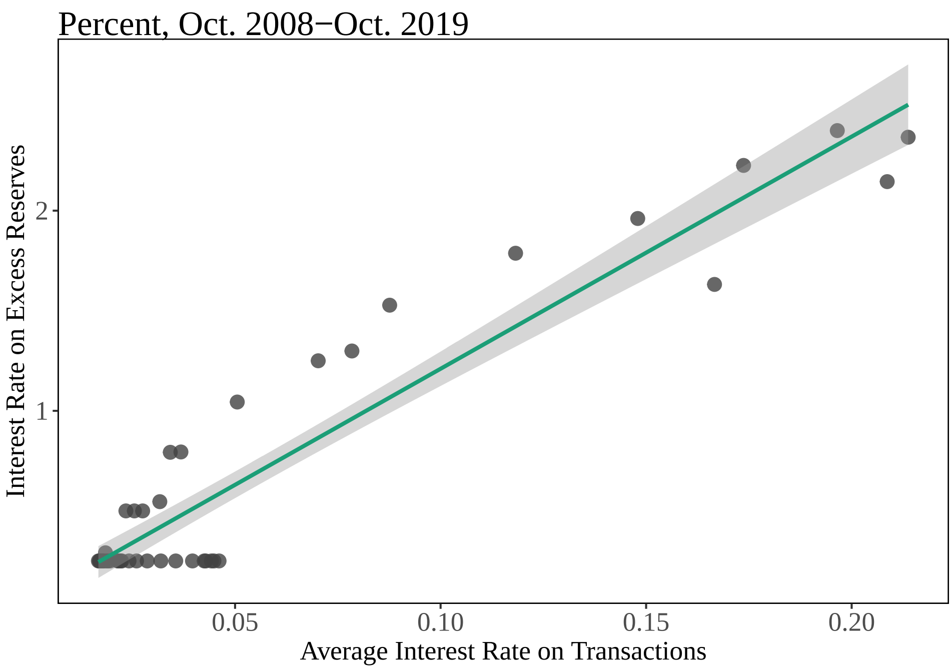

Figure 2: Interest Rates

Source: Federal Reserve Board and Chiu et al. (2019).

5 Bank deposits, interest-bearing money, and an illiquid bond market

In this section, I assume that the shock on consumer type, $\omega_t(i)$, is not publicly observable upon realization. In other words, household types are private information. Since there is no record-keeping, a welfare-improving transfer policy is infeasible, as type l consumers will misrepresent themselves as type h consumers. The optimal transfer policy would then essentially be a zero transfer (Andolfatto (2011)). In what follows, I introduce an illiquid bond in the monetary economy, that is subject to news shocks.

The central bank now issues two intrinsically worthless tokens, money and bonds, denoted by M and O, respectively. During the day, new bonds are issued at the discount price $0 < \alpha \leq 1$. Bonds are redeemed at par for money on the following day, and hence represent risk-free claims to future money. Since bonds are illiquid, they cannot be used to make payments. Instead, they can be exchanged for money in a secondary market at a competitive price α_2. I assume that this secondary market opens and closes before the news shock is realized, so that α_2 is independent of η. I also assume that this market opens right after the shock on consumer preferences is realized and closes before households travel to their respective locations.[15]

Money supply now evolves according to the central bank budget constraint, $M^+ - R^M M = O - \alpha O^+$. By assuming a constant bond-money ratio $\chi \equiv O/M > 0$, the budget constraint can be expressed as

$$\mu = \frac{R^M + \chi}{1 + \alpha\chi}. \tag{61}$$

Clearly, a zero discount policy ($\alpha = 1$) and zero nominal interest rate on money ($R^M = 1$) imply $\mu = 1$. In what follows, I will describe the respective optimization problems of the household for the day and the night.

5.1 Decision-making of households

As before, the entrepreneur's problem is unaffected. Since all bonds issued during the day will be redeemed into money at par, the composition of a money-bond portfolio during the day is irrelevant. This implies that the bank's problem remains unaffected, as total real money balances is all that really matters.

5.1.1 The day market

Let o denote the real bond holdings purchased by a household during the day. The household's choice problem is given by

$$W(d, a, z) \equiv \max_{s \geq 0, q \geq 0, o \geq 0} \left\{ \psi_1(z)d - \psi_1(z)s + R^M a - (q + \alpha o) + E_\eta V(s, q, o, \eta) \right\}. \tag{62}$$

The demand for real deposits s, real money demand q, and real bond demand o are characterized by

$$\psi_1(z) = E_\eta \frac{\partial V(s, q, \eta)}{\partial s}, \tag{63}$$

$$1 = E_\eta \frac{\partial V(s, q, \eta)}{\partial q}, \tag{64}$$

$$\alpha = E_\eta \frac{\partial V(s, q, \eta)}{\partial o}. \tag{65}$$

Note that the same envelope conditions (41) and (42) apply.

[15]This restriction on bond liquidity is what makes bonds essential in improving welfare when the added friction of private information is integrated into the environment.

5.1.2 The night market

Households enter the night market with a portfolio (d, q, o) when news is η. The bond market opens right after the consumer preference shock is realized. Subsequently, there is a news shock on the entrepreneurs' productivity right after the bond market closes. Let o_j denote the quantity of real bonds sold (where $o_j < 0$ denotes a purchase of real bonds) by a type j household in the bond market. Because the quantity of bonds sold cannot exceed the quantity available, there is a trading restriction on bond sales; in particular,

$$o_j \leq o. \tag{66}$$

Consumers with liquidity needs, may only purchase output at night by using either money or deposits. The deposit constraint (14) remains unaffected, but each consumer now faces the following cash constraint:

$$c_j^m \leq \frac{\phi_2(\eta)}{\phi_1} \left(q + \alpha_2 o_j\right). \tag{67}$$

The consolidated consumer debt-constraint now becomes

$$c_j \leq \psi_2(\eta) s + \frac{\phi_2(\eta)}{\phi_1} \left(q + \alpha_2 o_j\right), \tag{68}$$

with the evolution of real balances,

$$a_j^+ = \frac{\phi_1^+}{\phi_1} \left(q + \alpha_2 o_j + \frac{\phi_1}{\phi_2(\eta)} \left(y_j^m - c_j^m + o - o_j\right)\right).$$

For a household with realized consumer type $j \in \{l, h\}$, the choice problem is given by

$$
\begin{aligned}
V_j(s,q,o,z) \equiv \max_{c_j^d,c_j^m,y_j^d,y_j^m} \Bigg\{ & \omega_j u(c_j) - v(y_j) \\
& + \beta \mathrm{E}\Bigg[W\Bigg(\frac{1}{\psi_2(\eta)} (\psi_2(\eta)s + y_j^d - c_j^d), \\
& \qquad \frac{\phi_1^+}{\phi_1}\left(q + \alpha_2 o_j + \frac{\phi_1}{\phi_2(\eta)}\left(y_j^m - c_j^m + o - o_j \right)\right), z^+ \Bigg) \Bigg| \eta \Bigg] \\
& \qquad + \varepsilon_j (o - o_j) \\
& \qquad + \lambda_j(\eta) \left[\frac{\phi_2(\eta)}{\phi_1}(q + \alpha_2 o_j) + \psi_2(\eta)s - (c_j^d + c_j^m) \right] \Bigg\}, \quad (69)
\end{aligned}
$$

where $\varepsilon_j \geq 0$ is a Lagrange multiplier associated with constraint (66), and $\lambda_j(\eta) \geq 0$ is a Lagrange multiplier associated with the consolidated consumer debt-constraint (68).

Once again, assuming the no-arbitrage condition, the total supply of output at night is characterized by (46). The total consumption at night is characterized by

$$
\begin{aligned}
\omega_j u'(c_j(\eta)) &= \tfrac{\beta R^M \phi_1^+}{\phi_2(\eta)} + \lambda_j(\eta) && \text{if } \psi_2(\eta)s + \tfrac{\phi_2(\eta)}{\phi_1}\left(q + \alpha_2 o_j\right) \geq c_j(\eta) \\
c_j(\eta) &= \psi_2(\eta)s + \tfrac{\phi_2(\eta)}{\phi_1}\left(q + \alpha_2 o_j\right) && \text{otherwise.}
\end{aligned} \quad (70)
$$

The first-order condition with respect to unsold bond holdings yields

$$
\beta R^M \left[\frac{\phi_1^+}{\phi_1}\alpha_2 - \frac{\phi_1^+}{\phi_2(\eta)} \right] + \alpha_2 \frac{\phi_2(\eta)}{\phi_1}\lambda_j(\eta) = \varepsilon_j. \quad (71)
$$

Combining the latter two expressions gives

$$
\varepsilon_j = \frac{\phi_2(\eta)}{\phi_1}\alpha_2 \omega_j u'(c_j(\eta)) - \beta R^M \frac{\phi_1^+}{\phi_2(\eta)}. \quad (72)
$$

Another envelope condition in addition to (48) and (49) is

$$
\frac{\partial V_j(s,q,o,z)}{\partial o} = \frac{\phi_2(\eta)}{\phi_1}\alpha_2 \omega_j u'(c_j(\eta)). \quad (73)
$$

5.2 Equilibrium

In addition to the market-clearing conditions in (50), the bond market-clearing conditions are

$$o = \chi q$$
$$o_l + o_h = 0. \tag{74}$$

Note that $\partial V(s,q,o,z)/\partial o = 0.5 \partial V_l(s,q,o,z)/\partial o + 0.5 \partial V_h(s,q,o,z)/\partial o$. The latter expression combined with condition (65) imply that $\alpha_2 = \alpha$. Moreover, using the envelope condition (73), once again we have the same restriction as (56). It can be easily shown that type l households will use their money to buy bonds and type h households will sell their bonds for money; that is, $o_l < 0 < o_h$. If the consolidated debt-constraint for type l consumers is slack (so that $\lambda_l(\eta) = 0$), then $u'(c_l(\eta)) = v'(y(\eta))$. This together with a slack bond-sales constraint for type l consumers ($\varepsilon_l = 0$) yields

$$\frac{\phi_2(\eta)}{\phi_1}\alpha u'(c_l(\eta)) = v'(y(\eta)) = \beta R^M \frac{\phi_1^+}{\phi_2(\eta)}. \tag{75}$$

Assuming stationarity, combining the latter expression with (56) leads to

$$\frac{\alpha\mu}{R^M} = \beta\left[\pi A(y(b)) + (1-\pi)A(y(g))\right]. \tag{76}$$

Note the similarity between the above expression and condition (60).

Condition (76) and (26) derived earlier, characterize the competitive equilibrium in which bank deposits, interest-bearing money, and illiquid bond are valued. Also, from condition (76), implementation of a first-best allocation will require a policy that satisfies $\alpha\mu = \beta R^M$. This is a case when the bond market supplies the agents with sufficient liquidity, as the ability of the government to repay its debt means that bonds may generally be accepted in exchange for money to meet the different liquidity needs of agents. Agents adjust their asset portfolio and liquidity is channeled from bond buyers (type l consumers) to bond sellers (type h consumers). Assuming that the policy $\alpha\mu = \beta R^M$ is satisfied, then together with the central bank budget constraint (61), implies

$$\mu = R^M + (1-\beta R^M)\chi. \tag{77}$$

Observe that the implied inflation rate is strictly positive for any $\chi > 0$ and $R^M \geq 1$. Clearly, this policy restriction requires the discount rate $\alpha = \beta R^M/\mu < 1$. Furthermore, an increase in the bond-money ratio is associated with a higher nominal interest rate.

I will now check if the bond-sales constraint for type h consumers will bind or not.

Contrary to the literature, since the consumer debt-constraint is influenced by news, the question of whether this constraint binds or not is not entirely determined by the bond discount price α or the nominal interest rate. Suppose the debt-constraint for type h consumers is slack, then $\delta u'(c_h(\eta)) = \beta R^M \phi_1^+ / \phi_2(\eta)$. If type-$h$ bond-sales constraint (66) is also slack then this together requires $\phi_1 \leq \phi_2(\eta)$ for $\alpha \leq 1$.

First, consider the case $o_h = o$ and suppose the debt-constraint for type h binds, that is, $\lambda_h > 0$. Invoking (74) yields

$$c_h(\eta) = \psi_2(\eta)s + \frac{\phi_2(\eta)}{\phi_1}(q + \alpha\chi q).$$

Assuming a binding debt-constraint for type l and once again using the market-clearing conditions, we can solve for q and rewrite the following restriction on policy variables α and χ:

$$c_h(\eta) = (1 + \alpha\chi)y(\eta) - \alpha\chi\psi_2(\eta)S. \tag{78}$$

Note that the binding bond-sales constraint for type h consumers implies $\varepsilon_h > 0$. At the same time, if both the bond-sales constraint and the debt-constraint for type l consumers bind, then it is impossible for type h to have a binding bond-sales constraint.

Lemma 4 *The bond-sales constraint for type h consumers cannot bind tightly in a news economy.*

Proof. Since $\varepsilon_h > \varepsilon_l = 0$, (72) reveals that $\delta u'(c_h(\eta)) > u'(c_l(\eta))$. Also, if the cash constraint for type l consumers is slack then $\delta u'(c_h(\eta)) > v'(y(\eta))$. This implies $A(y) > 1$ and using (76), $\alpha > \beta R^M / \mu > 1$. But this is a contradiction; as $\alpha \leq 1$. ■

Now, consider the case $o_h < o$. Then the slack bond-sales constraint for type h consumers means $\varepsilon_h = 0$. If $\varepsilon_h = \varepsilon_l = 0$, then $u'(c_l(\eta)) = \delta u'(c_h(\eta))$. Owing to $\lambda_l = 0$ yields $\delta u'(c_h(\eta)) = v'(y(\eta))$. Clearly, $A(y^*) = 1$ entails a policy that satisfies $\alpha\mu = \beta R^M$. Substituting $\alpha = \beta R^M / \mu$ into (78), where μ is given by (77), leads to

$$c_h(\eta) = \frac{(R^M + \chi)y(\eta) - \beta R^M \chi\psi_2(\eta)S}{R^M + (1 - \beta R^M)\chi}. \tag{79}$$

The expression above implies that there exists a $\chi^* > 0$, $R^{M^*} > 1$, and $\mu^* > 1$ that can implement the first-best allocation y^*, so that the bond-sales constraint of type h consumers remains slack. We have the following proposition.

Proposition 6 *i) In a news economy with bank deposits, interest-bearing money, and illiquid* *bond, if $\phi_1 < \phi_2(\eta)$ then the efficient allocation is implementable for any bond-money ratio $0 < \chi^* \leq \chi < \infty$ and money growth rate $\mu^* = R^M + (1 - \beta R^M)\chi > 1$, with an associated nominal interest rate $R^{M^*} = \beta^{-1} > 1$ and a discount rate $\alpha^* < 1$. The lending market may remain suboptimal with $p < p^*$.*

ii) In a no-news economy, there is no monetary equilibrium when policy is restricted to $0 < \chi^* \leq \chi < \infty$, $\mu^* = R^M + (1 - \beta R^M)\chi > 1$, with an associated $R^{M^*} = \beta^{-1} > 1$ and $\alpha^* < 1$; most importantly, when $p = p^*$.

Proof. Since $A(y^*) = 1$, condition (76) is satisfied with $\alpha^*\mu^* = \beta R^{M^*}$. Now we will need to check the condition that can guarantee positive bond balances and concurrently satisfy $\psi_1(z) > 0$. Substituting $c_l(\eta)$ and $c_h(\eta)$ into (70) and then using the market-clearing conditions (50) and (74) imply

$$o \geq \frac{\chi^*\mu^*}{\beta R^{M^*}} \left[y^* v'(y^*) - \beta z(b) f'(p) S\right]$$

$$\text{or, } o \geq \frac{\chi^*\mu^*}{\beta R^{M^*}} \left[y^* v'(y^*) - \beta R^L S\right],$$

where $R^{L^*} < R^L$ and $p < p^*$ by assuming binding constraints for the entrepreneurs and the banks. As long as $y^* v'(y^*) > \beta R^L S = \beta z(b) f'(p) S$, $o > o_h > 0$ and the bond-sales constraint for type h consumers will remain slack (following Lemma 4). Consequently, any value $o < \infty$ satisfying the latter inequalities is an equilibrium. Similar to the proof in Proposition 4, if $y^* v'(y^*) = \beta z^e f'(p^*) S = \beta R^{L^*} S$ then $o = 0$, which violates our assumption of $o_l < 0 < o_h < o$. ■

Having a limit on bond holdings for type h consumers means that type h cannot get sufficient liquidity on bond sales. An optimal allocation of liquidity between the two types of consumers requires the bond market to generate sufficient liquidity against news shocks. An insufficient liquidity from bond sales is infeasible, especially with news shocks creating an additional liquidity shortage. As a consequence, the coexistence of government debt instruments with other private assets requires the sufficiency of liquidity provision. With the exception of strictly positive inflation with illiquid bonds, Proposition 6, by and large, replicates the result achieved by the optimal type-contingent transfer policy described in Proposition 4. The illiquid bond helps achieve socially desirable allocations, even though household preferences are unknown to the central bank. The lending market may still remain suboptimal for the same aforementioned reasons. However, illiquid bond and interest-bearing money cannot coexist with bank deposits in the no-news case. The reasoning is the same as what was stated earlier; namely, that there is no need for government intervention if the first-best solution can be achieved with private money.

In the no-news case, both interest-bearing money and illiquid bond are not essential.

In the next subsection, I investigate the possibility of rendering the short-run rate of return on interest-bearing money insensitive to news by restricting the use of bank deposits as private money.

5.3 Illiquid bank deposits

I assume now that bank deposits are illiquid, that is, they cannot be used to make payments at night. In this case, only interest-bearing money can be used to make payments at night. This type of a cash-in-advance constraint is frequently imposed in the literature. I will show how a cash-in-advance constraint of this form is welfare-enhancing.[16]

The choice problem of the agents in the day is unaffected, but the decisions on consumption and production at night will clearly change. The supply of night output y is still characterized by condition (46). However, the no-arbitrage condition (45) is not relevant in this case, as fiat money can only be used for the purchase of goods in the night. Adding the constraint $s = 0$ means that the desired consumption at night is characterized by

$$\begin{aligned} \omega_j u'(c_j(\eta)) &= \tfrac{\beta R^M \phi_1^+}{\phi_2(\eta)} + \lambda_j(\eta) \qquad \text{if } \tfrac{\phi_2(\eta)}{\phi_1}\left(q + \alpha_2 o_j\right) \geq c_j(\eta) \\ c_j(\eta) &= \tfrac{\phi_2(\eta)}{\phi_1}\left(q + \alpha_2 o_j\right) \qquad\qquad \text{otherwise.} \end{aligned} \tag{80}$$

I anticipate that the cash-in-advance constraint will bind for a growing supply of money ($\mu \geq 1$), a positive interest rate $R^M \geq 1$, a bond-money ratio $\chi > 0$, and a bond discount price $\alpha \leq 1$. Applying the market-clearing conditions and combining (50) and (70), the equilibrium value of money in the night is expressed as

$$\phi_2(\eta) = \frac{\phi_1 y(\eta)}{Q} - 2\phi_1\left(\frac{R^M}{\mu} - 1\right).$$

Together, these restrictions lead to

$$v'(y(\eta)) = \frac{\beta R^M \phi_1}{\mu\left[\frac{y(\eta)}{M^+} - 2\phi_1\left(\frac{R^M}{\mu} - 1\right)\right]}.$$

Clearly, this implies that the equilibrium level of night output is independent of news, that is, $y = y(\eta)$. This is because payments at night are now solely made with a risk-free asset. Then

[16] A result that has also been pointed out in Lagos and Rocheteau (2008), where placing an exogenous restriction to render capital less liquid generates a demand for outside money, so that such a restriction is indeed welfare improving.

solving for ϕ_1 results in the equilibrium restriction

$$\phi_1 = \frac{\mu v'(y)y/M^+}{\beta R^M + 2\mu v'(y)\left(\frac{R^M}{\mu} - 1\right)}. \tag{81}$$

Substituting the value of money in the day and night into (56), we have the equilibrium restriction

$$\frac{\alpha\mu}{R^M} = \beta A(y). \tag{82}$$

Conditions (81) and (82) characterize the equilibrium pair (ϕ_1, y). Furthermore, we can achieve a first-best allocation with the given policy below.

Proposition 7 *In a no-news economy, rendering bank deposits illiquid by imposing a cash-in-advance constraint at night improves welfare with the given policy $\alpha\mu = \beta R^M$. Interest-bearing money and an illiquid bond can coexist with bank deposits when $p = p^*$.*

Proof. Since $A(y^*) = 1$, the restriction (82) is satisfied when $\alpha\mu = \beta R^M$. ■

In contrast to Andolfatto and Martin (2013), this paper finds that imposing a cash-in-advance constraint to make bank deposits less liquid clearly enhances social welfare in the absence of news. In their study, such a constraint actually diminishes welfare when there is no news. This discrepancy might arise from their omission of private information regarding consumer types. Conversely, in my model, factoring in private information about consumer liquidity shocks necessitates an illiquid bond to broaden the scope of trades that improve welfare. Here, a cash-in-advance is a trading restriction on bank deposits that is designed to improve welfare. This type of result has been highlighted in Andolfatto (2011) and Kocherlakota (2003), where restricting the liquidity properties of bonds improves allocative efficiency. Without the friction of private information on consumer preferences, a cash-in-advance may restrict trading opportunities. This could explain the discrepancies between my result and the result in Andolfatto and Martin (2013). And as consequence, a cash-in-advance constraint in a no-news economy eliminates the suboptimality in the lending market that may exist with news; so that $p = p^*$. The restriction of using bank deposits as a means of payment reduces currency competition and fiat money is no longer linked by an arbitrage condition to the price of private money. The value of interest-bearing money becomes insensitive to news or information and its average rate of return is independent of news shocks. Most importantly, interest-bearing money and an illiquid bond can coexist with bank deposits when there are no news shocks.

5.3.1 Numerical Example

To see how the model works with a cash-in-advance constraint on bank deposits in a news economy with interest-bearing money and an illiquid bond, I provide some numerical examples. For functional forms, I assume $u(c) = {(c^{1-\sigma}-1)}/{(1-\sigma)}$, $v(y) = y$, and $f(p) = p^{\theta}$, which implies $c_l^* = 1$, $c_h^* = \delta^{1/\sigma}$, and $y^* = {(1+\delta^{1/\sigma})}/{2}$, and $x^* = 1/\{(\beta\theta)[\pi z(b)+(1-\pi)z(b)]\}^{1/(\theta-1)}$. Given linear preferences in the day, I consider the average consumption in the day. Period expected utlity or welfare in the cash-in-advance economy is

$$W^{\text{CIA}} = f(p) - p + \frac{1}{2}u(c_l) + \frac{1}{2}\delta u(c_h) - v(y).$$

Similarly, welfare in the news economy is

$$\begin{aligned} W^{\text{News}} &= f(p) - p + \frac{\pi}{2}\left[u(c_l(b)) + \delta u(c_h(b))\right] \\ &+ \frac{(1-\pi)}{2}\left[u(c_l(g)) + \delta u(c_h(g))\right] \\ &- \left[\pi v(y(b)) + (1-\pi)v(y(g))\right]. \end{aligned}$$

Using the functional forms and condition (82), the equilibrium allocation in the cash-in-advance economy is as follows

$$y^{\text{CIA}} = \left[\frac{\beta\delta R^M}{2\alpha\mu - \beta R^M}\right]^{\frac{1}{\sigma}},$$

$$c_h^{\text{CIA}} = \left[\frac{\left(R^M + \chi\right)}{R^M + \left(1 - \beta R^M\right)\chi}\right]\left[\frac{\beta\delta R^M}{2\alpha\mu - \beta R^M}\right]^{\frac{1}{\sigma}},$$

$$c_l^{\text{CIA}} = \left[\frac{\beta\delta R^M}{2\alpha\mu - \beta R^M}\right]^{\frac{1}{\sigma}}\left[\frac{R^M + \chi - 2\beta R^M\chi}{R^M + \left(1 - \beta R^M\right)\chi}\right].$$

For the good state, fix $c_l(g) = c_l^*$, $c_h(g) = c_h^*$, and $y(g) = y^*$. Then using (76), the equilibrium allocation in the bad state is characterized by

$$y(b) = \frac{\delta^{\frac{1}{\sigma}}}{\left[\frac{2\alpha\mu}{\beta\pi R^M} - \frac{1-\pi}{\pi}\frac{2^{\sigma}\delta}{(1+\delta^{\frac{1}{\sigma}})^{\sigma}} - \frac{1-2\pi}{\pi}\right]^{\frac{1}{\sigma}}},$$

$$c_h(b) = \frac{\delta^{\frac{1}{\sigma}} \left(R^M + \chi\right)}{\left[R^M + \left(1 - \beta R^M\right)\chi\right] \left[\frac{2\alpha\mu}{\beta\pi R^M} - \frac{1-\pi}{\pi} \frac{2^\sigma \delta}{(1+\delta^{\frac{1}{\sigma}})^\sigma} - \frac{1-2\pi}{\pi}\right]^{\frac{1}{\sigma}}},$$

$$c_l(b) = \frac{\delta^{\frac{1}{\sigma}} \left(R^M + \chi - 2\beta R^M \chi\right)}{\left[R^M + \left(1 - \beta R^M\right)\chi\right] \left[\frac{2\alpha\mu}{\beta\pi R^M} - \frac{1-\pi}{\pi} \frac{2^\sigma \delta}{(1+\delta^{\frac{1}{\sigma}})^\sigma} - \frac{1-2\pi}{\pi}\right]^{\frac{1}{\sigma}}}.$$

For parameters, I assume $\beta = 0.95$, $\delta = 100$, $\sigma = 10$ and $\theta = 0.02$. With these parameters, the first-best allocation is $c_l^* = 1$, $c_h^* = 1.58$ and $y^* = 1.29$. First, I fix $\alpha = 0.48$, $O = 2$, $M = S = 10$, $R^M = 1.01$, $\pi = 0.25$, $z(b) = 0$ and illustrate how changing $z(g)$ affects the allocation in the competitive equilibrium of the cash-in-advance economy and the news economy, respectively.[17] Then, I compute and compare the welfare in these respective economies. Pick $z(g) = 0.0001$ so that condition (31) just about satisfied.[18] The equilibrium allocation in the cash-in-advance economy is $p^{CIA} = p^* = 1.08$, $c_l^{\text{CIA}} = 1.61$, $c_h^{\text{CIA}} = 2.36$ and $y^{\text{CIA}} = 1.99$. Welfare in the cash-in-advance economy is 4.38. On the other hand, the equilibrium allocation in the news economy is $p^{News} = p^* = 1.08$, $c_l(b) = 0.95$, $c_h(b) = 1.39$, $c_l(g) = c_l^* = 1$, $c_h(g) = c_h^* = 1.58$ and $y(b) = 1.17$, $y(g) = y^* = 1.29$. Welfare in the news economy is 4.91. For this parameterization, the difference in welfare between the cash-in-advance economy and the news economy is -0.53. Now consider $z(g) = 0.5$. Then the equilibrium allocation in the cash-in-advance economy is $p^{CIA} = p^* = 0.006$, with the night-allocation mostly unchanged. Welfare in the cash-in-advance economy now increases slightly. Similar result applies for the equilibrium quantities and welfare in the news economy. Most importantly, the difference in welfare between these two respective economies now increases by a small margin. As expected, an increase in the magnitude of the good state, represented by $z(g)$, implies that the news economy outperforms the cash-in-advance economy in terms of welfare, although only marginally. This has been tested by creating a grid of $z(g)$ that takes values between 0.0001 and 0.5. Overall, 1000 grid points of $z(g)$ had been generated between the first and last elements of the grid.

I now fix $z(g) = 0.0001$ and vary the nominal interest rate, R^M, to better understand the welfare effects of restricting bank deposits as a means of payment. Let $\Delta(R^M)$ represent the welfare difference between the cash-in-advance and news economies as a function of the nominal interest rate. I create a range of 1000 R^M values between 1 and 10. For each R^M, I compute the welfare in both economies and calculate the welfare difference, $\Delta(R^M)$, for the overlapping values. This calculation is repeated 1000 times for each R^M value in the range. Moreover, for

[17] Note that the money growth rate μ is determined by the budget constraint (61) once the nominal interest rate, the bond discount price, and the bond-money ratio are given.

[18] In fact, this is the minimum value of $z(g)$ so that the deposit constraint in the good state is barely slack.

a fixed nominal interest rate, $R^M > 1$, money supply, bond supply, and $z(b)$, I determine how many parameterizations of $\{\alpha, \beta, \delta, \sigma, \theta, \pi, z(g)\}$ satisfy the condition $\Delta(R^M) > 0$. After a total of $10,000,000$ iterations, there are $11,404,000$ parameter combinations for which $\Delta(R^M)$ is strictly positive. This suggests that we can construct policies such that restricting the liquidity of bank deposits may improve welfare, although this is not universally the case. The findings of this numerical exercise can be summarized in the following result:

Proposition 8 *In a news economy, rendering bank deposits illiquid by imposing a cash-in-advance constraint at night has an ambiguous welfare effect.*

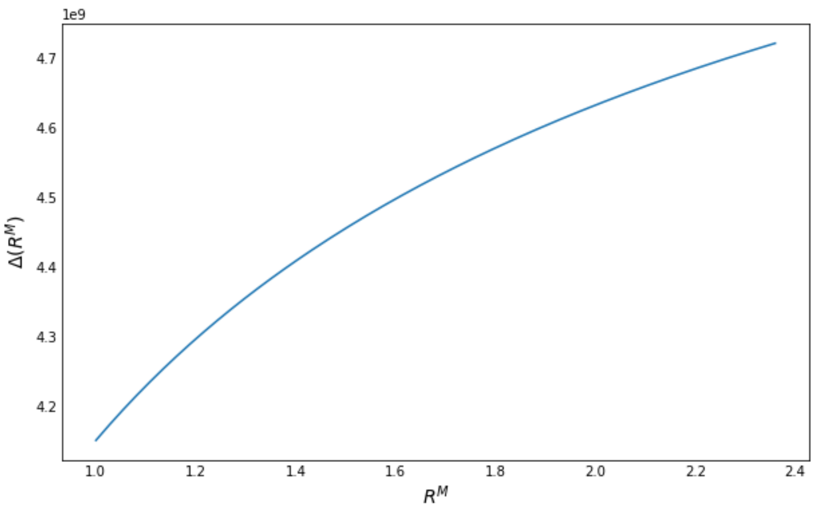

Figure 3: Welfare gain from restricting bank deposits as a function of R^M

6 Conclusion

If private currencies were not subject to information and limited commitment frictions, there would be no reason why they shouldn't be used as payment instruments. Historically, they were widely used before the dominance of government-issued fiat money. The problem lies when these private currencies are backed by other productive assets. In the model, banks issue private currencies in the form of bank deposits, which circulate as exchange media. The price of bank deposits may fluctuate excessively in response to news events surrounding technological innovations of the firms' future output, which is pledged as collateral by the firms to obtain loans from the banks. The price volatility of bank deposits does not inherently inhibit their use as exchange media, as long as the supply of deposits is not scarce. Individuals with higher liquidity needs are still willing to pay a premium for intertemporal gains to trade. The problem emerges when bad news (although socially irrelevant) results in binding debt-constraints, which can lead to a liquidity shortage. If bank deposits were not used as a medium of exchange, their price fluctuations might be benign. However, when they play a role in the payments system, any extraneous information can lead to excessive volatility in deposit prices, particularly when there is a liquidity shortage due to limited commitment.

To a degree, the adverse impact of private currencies might explain the widespread use of government-issued fiat money. While private banks might use asset tranching from an existing asset pool to create informationally insensitive exchange media, this approach faces significant limitations if there is an initial asset scarcity. Non-disclosure practices might help mitigate short-term asset price fluctuations. But when tranching or non-disclosure is not viable, central banks can provide high-quality debt instruments to overcome the shortcomings of the highly price sensitive nature of private currencies.

If a lump-tax instrument is not available, then the central bank can conduct a type-contingent transfer policy to restore efficiency. This is assuming if individual preferences over desired consumption needs are public information. Interest-bearing money provides the central bank with an additional policy tool that can positively impact depressed asset prices. Banks can invest in interest-bearing reserves as a hedge against risk shocks, enabling the central bank to influence asset prices through an investment channel. In particular, a welfare-improving policy entails a positive inflation rate and a strictly positive nominal interest rate. The fact that interest-bearing money is not backed by any productive asset allows the central bank to effectively create assets by issuing debt.

When the added friction of private information over desired consumption needs is incorporated into the model, an illiquid bond becomes essential. As before, first-best implementation is also feasible through a well-designed policy that permits some inflation and a strictly positive nominal interest rate. By imposing a cash-in-advance constraint, government debt instruments can become insensitive to news and coexist with private currencies under certain conditions. Of course, this only works to the extent that the central bank is willing to maintain low levels of inflation.

The model framework is sufficiently simple to allow for many interesting extensions. One such extension could involve allowing entrepreneurs to default on their debt obligations and banks to fail, with the banks then paying premiums to a deposit insurance agency. Additionally, introducing a meaningful role for equity finance, as detailed in Dermine (1986), could be explored. Another concept is to include scenarios where banks invest in riskier projects to gain access to cheaper funding. Examining the impact of banks possessing some degree of market power, especially in the context of information frictions, is also a realistic and worthwhile endeavor.

References

Amendola, N., L. Carbonari, L. Ferraris, et al. (2021). Three liquid assets. Technical report, Tor Vergata University, CEIS.

Andolfatto, D. (2010). Essential interest-bearing money. *Journal of Economic theory 145*(4), 1495–1507.

Andolfatto, D. (2011). A note on the societal benefits of illiquid bonds. *Canadian Journal of Economics/Revue canadienne d'économique 44*(1), 133–147.

Andolfatto, D. (2013). Incentive-feasible deflation. *Journal of Monetary Economics 60*(4), 383–390.

Andolfatto, D. (2021). Assessing the impact of central bank digital currency on private banks. *The Economic Journal 131*(634), 525–540.

Andolfatto, D., A. Berentsen, and C. Waller (2014). Optimal disclosure policy and undue diligence. *Journal of Economic Theory 149*, 128–152.

Andolfatto, D. and F. M. Martin (2009). Money and capital as competing media of exchange in a news economy. *Federal Reserve Bank of St. Louis Working Paper*.

Andolfatto, D. and F. M. Martin (2013). Information disclosure and exchange media. *Review of Economic Dynamics 16*(3), 527–539.

Berentsen, A., G. Camera, and C. Waller (2007). Money, credit and banking. *Journal of Economic theory 135*(1), 171–195.

Bernanke, B. S. and A. S. Blinder (1992). The federal funds rate and the channels of monetary transmission. *American economic review 82*(4), 901–21.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics 1*, 1341–1393.

Brunnermeier, M. K. and D. Niepelt (2019). On the equivalence of private and public money. *Journal of Monetary Economics 106*, 27–41.

Caballero, R. J. (2006). On the macroeconomics of asset shortages.

Champ, B. A. (2007). The national banking system: A brief history.

Chiu, J., S. M. Davoodalhosseini, J. Hua Jiang, and Y. Zhu (2019). Bank market power and central bank digital currency: Theory and quantitative assessment. *Available at SSRN 3331135*.

Choi, M. and F. Liang (2021). Learning and money adoption. *Available at SSRN 3632489*.

Christiano, L. J., R. Motto, and M. Rostagno (2014). Risk shocks. *American Economic Review 104*(1), 27–65.

Dang, T. V., G. Gorton, B. Holmström, and G. Ordonez (2017). Banks as secret keepers. *American Economic Review 107*(4), 1005–29.

Davies, G. (2010). *History of money*. University of Wales Press.

Dermine, J. (1986). Deposit rates, credit rates and bank capital: the klein-monti model revisited. *Journal of Banking & Finance 10*(1), 99–114.

Drechsler, I., A. Savov, and P. Schnabl (2017). The deposits channel of monetary policy. *The Quarterly Journal of Economics 132*(4), 1819–1876.

Fernández-Villaverde, J. and D. Sanches (2019). Can currency competition work? *Journal of Monetary Economics 106*, 1–15.

Goodfriend, M. and B. T. McCallum (2007). Banking and interest rates in monetary policy analysis: A quantitative exploration. *Journal of Monetary Economics 54*(5), 1480–1507.

Gu, C., H. Han, and R. Wright (2020). The effects of news when liquidity matters. *International Economic Review 61*(4), 1411–1435.

Gu, C., C. Monnet, E. Nosal, and R. Wright (2019). On the instability of banking and financial intermediation. Technical report, Department of Economics, University of Missouri.

Hirshleifer, J. (1971). The private and social value of information and the reward to inventive activity. *American Economic Review 61*(4), 561–574.

Hu, T.-W. (2021). Optimal monetary policy with interest on reserves and capital over-accumulation. *Journal of Economic Theory 196*, 105319.

Keister, T. and D. R. Sanches (2019). Should central banks issue digital currency?

Kiyotaki, N. and J. Moore (2019). Liquidity, business cycles, and monetary policy. *Journal of Political Economy 127*(6), 2926–2966.

Kocherlakota, N. R. (2003). Societal benefits of illiquid bonds. *Journal of Economic Theory 108*(2), 179–193.

Lagos, R. (2010). Asset prices and liquidity in an exchange economy. *Journal of Monetary Economics 57*(8), 913–930.

Lagos, R. and G. Rocheteau (2008). Money and capital as competing media of exchange. *Journal of Economic theory 142*(1), 247–258.

Lagos, R. and R. Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of political Economy 113*(3), 463–484.

Monnet, C., A. Petursdottir, and M. Rojas-Breu (2019). Central bank account for all: Efficiency and stability. Technical report, Mimeo.

Rocheteau, G., R. Wright, and C. Zhang (2018). Corporate finance and monetary policy. *American Economic Review 108*(4-5), 1147–86.

Wallace, N. (2014). Optimal money creation in "pure currency" economies: a conjecture. *The Quarterly Journal of Economics 129*(1), 259–274.

Williamson, S. D. (1999). Private money. *Journal of Money, Credit and Banking*, 469–491.

Williamson, S. D. (2019). Central bank digital currency: Welfare and policy implications.

Williamson, S. D. (2021). Central bank digital currency and flight to safety. *Journal of Economic Dynamics and Control*, 104146.

Xiang, H. (2013). Optimal monetary policy: distribution efficiency versus production efficiency. *Canadian Journal of Economics/Revue canadienne d'économique 46*(3), 836–864.